Physics 121 – October 26, 2017

Announcements:

Answers in back of book for #41 may not be correct.
We are only covering through section 10.5 this week
Interesting NRAO seminar on Friday, Nov 3.

Assignments:

This week:

- Read Chapter 10.
- Complete ETA Problem Set #10 by Monday, Oct 30.
- End-of-chapter problems: Chap 10 #35, 41, 54, 62, 67, and 68. Due by 4 pm, Oct 30.
- Recitation: Practice problems on springs, Chap 9, and on free body diagrams.

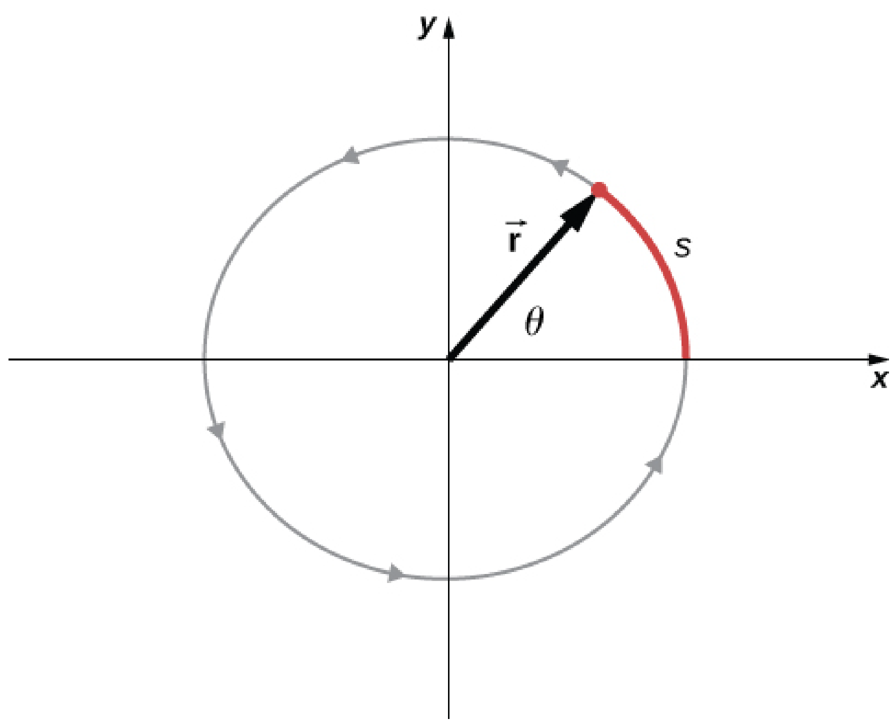
Topics for today:

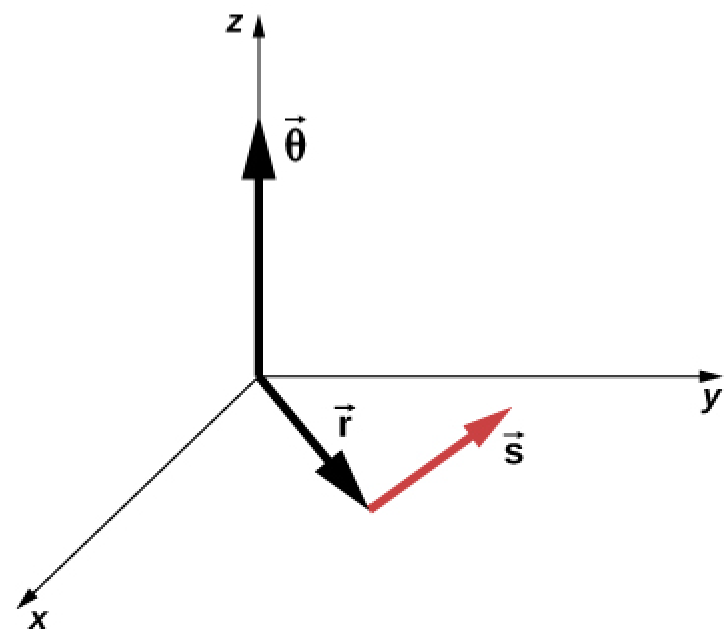
- Definitions for rotational motion: angular displacement, angular velocity, and angular acceleration.
- Kinematic equations for rotational motion with constant angular acceleration.
- Moment of inertia.
- Rotational kinetic energy and conservation of total energy.

Quantities for Describing Rotational Motion

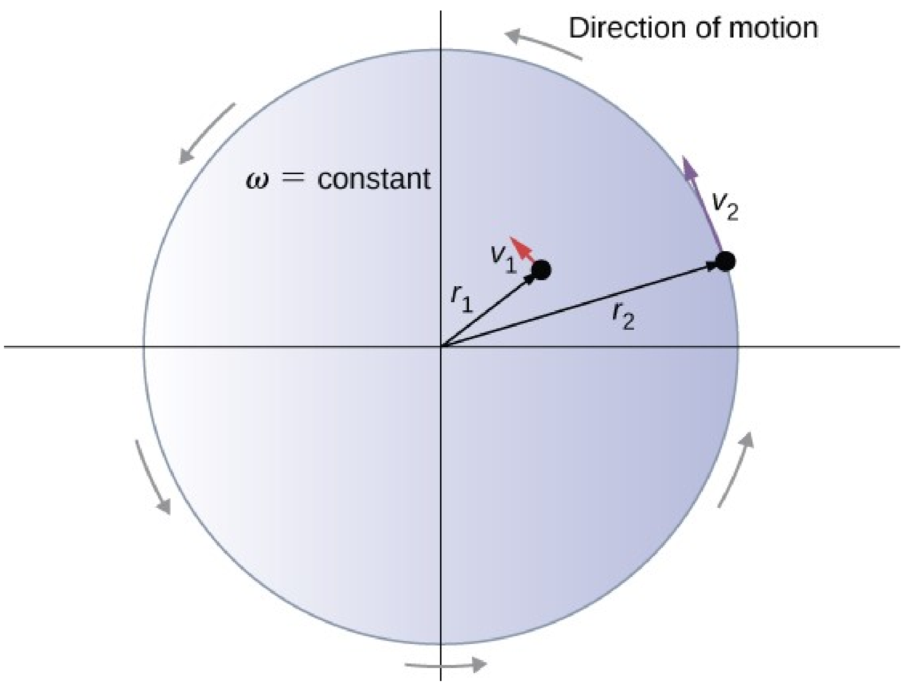
r = distance from rotational axis
θ = angular displacement
ω = angular velocity = dθ/dt
α = angular acceleration = dω/dt

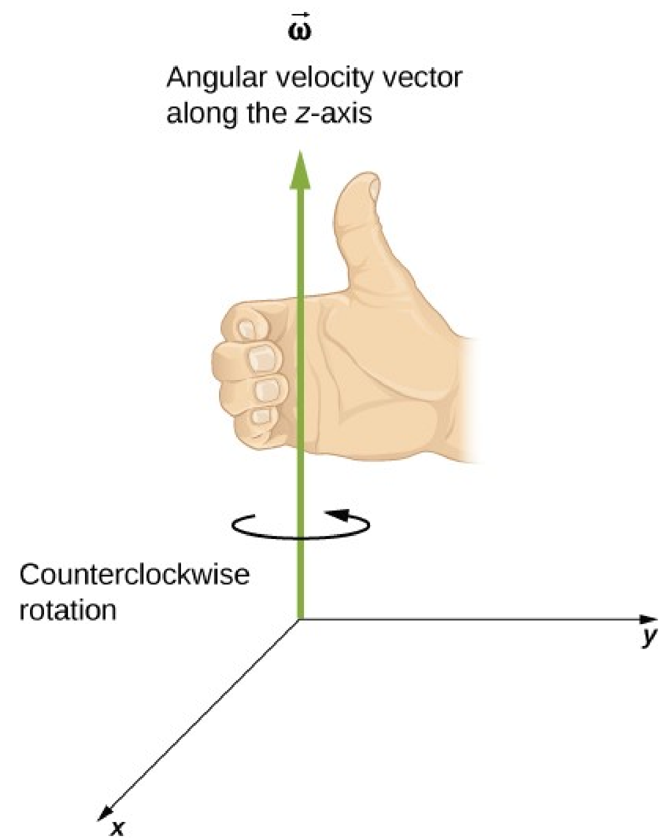
v = tangential velocity = rω
a = tangential acceleration = rα



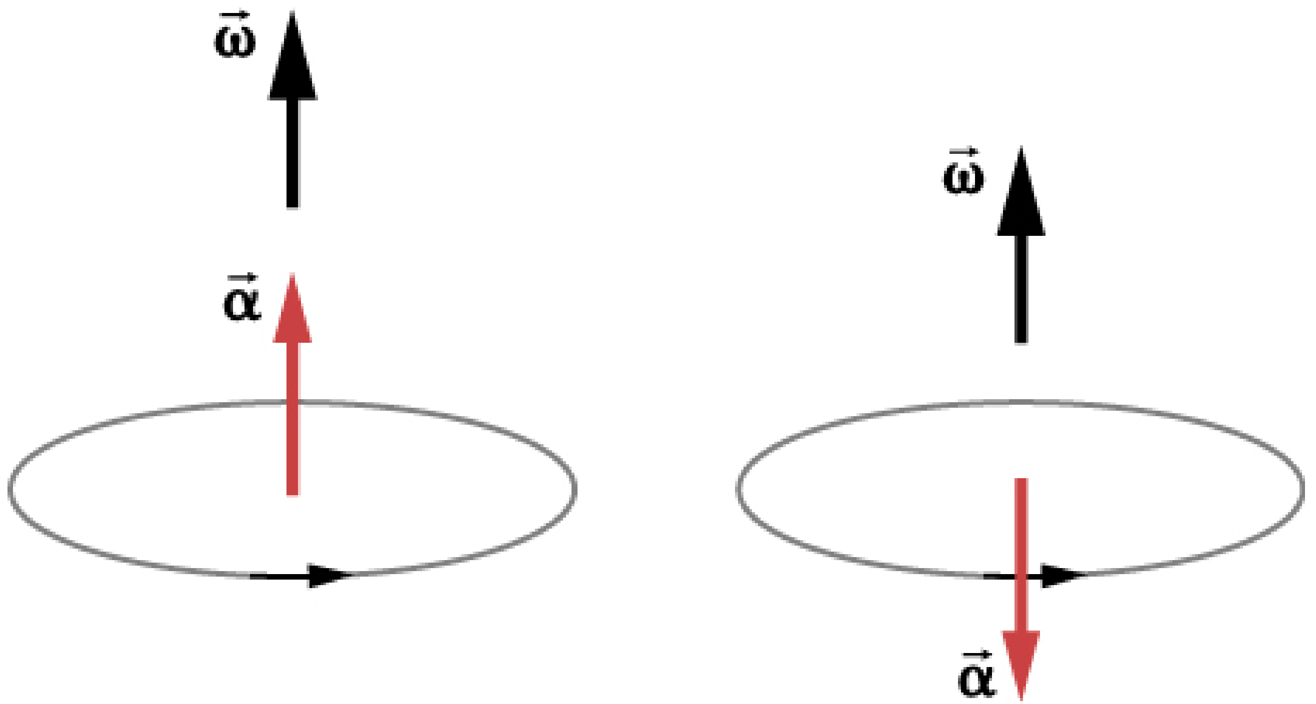


	Linear	Rotational
Position	x	θ
Velocity	$v = \frac{dx}{dt}$	$\omega = \frac{d\theta}{dt}$
Acceleration	$a = \frac{dv}{dt}$	$\alpha = \frac{d\omega}{dt}$





α will always be parallel to ω (if ω is increasing in magnitude with time) or anti-parallel to ω (if ω is decreasing with time).



(a) Rotation rate counterclockwise and increasing

(b) Rotation rate counterclockwise and decreasing

Acceleration in Uniform vs. Non-uniform Circular Motion

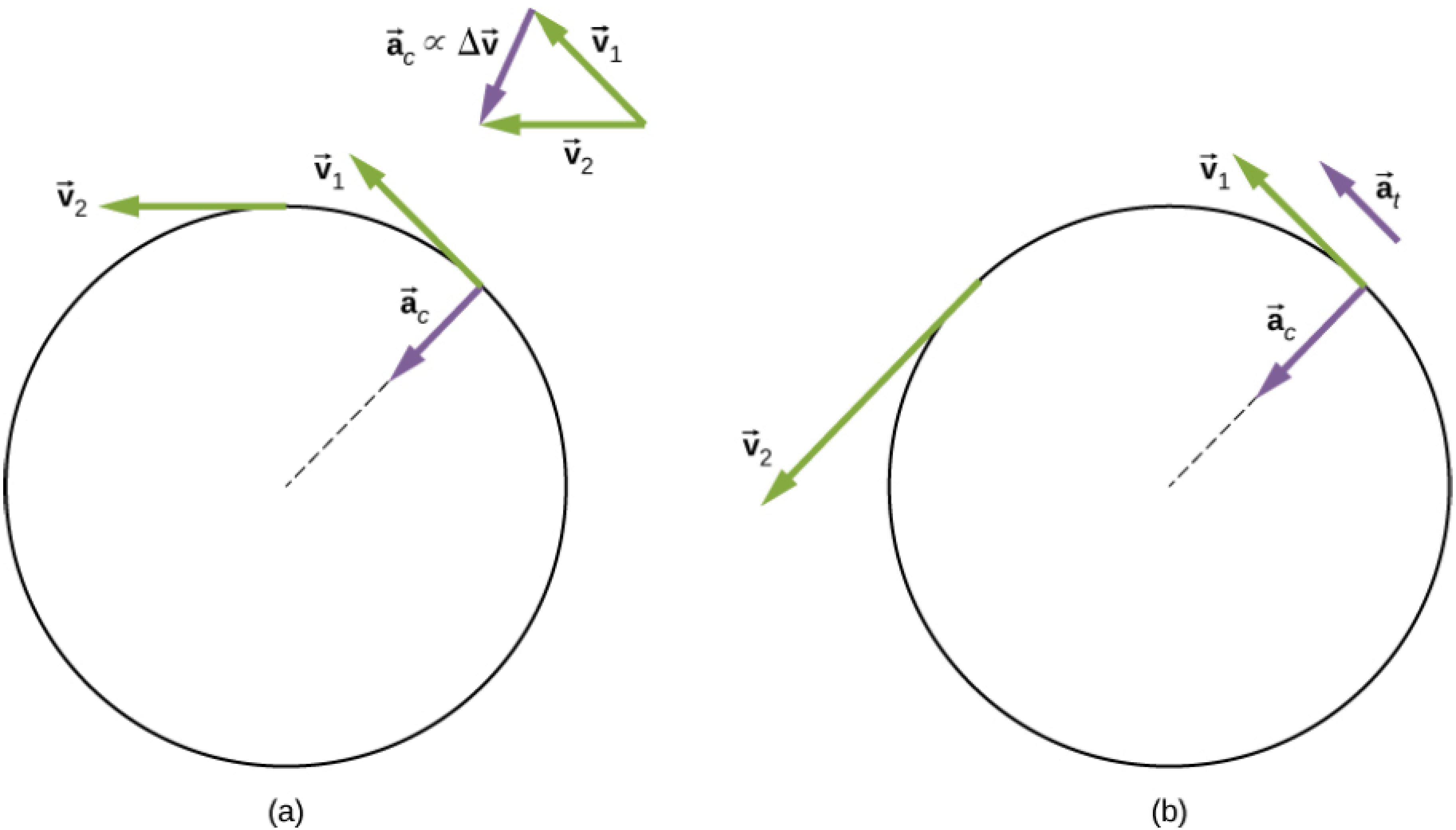
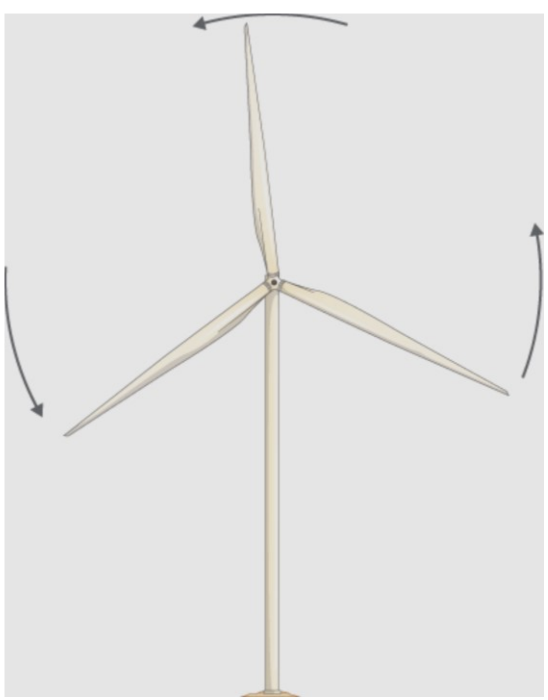


Figure 10.14 (a) Uniform circular motion: The centripetal acceleration a_c has its vector inward toward the axis of rotation. There is no tangential acceleration. (b) Nonuniform circular motion: An angular acceleration produces an inward centripetal acceleration that is changing in magnitude, plus a tangential acceleration a_t.

Example



Let's examine one of the turbines in the Brazos wind farm in west Texas. It has blades that are 40 meters long and the turbine is rotating at a constant 20 rpm (revolutions per minute).

What is the magnitude (in rad/s) and direction of ω ? What is the tangential velocity at the blade tip?

For constant angular acceleration, the kinematic equations look similar to the translational kinematic equations.

Relationships between Rotational and Translational Motion

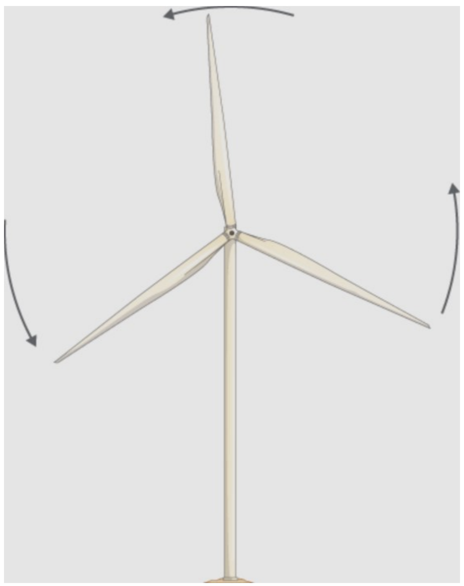
We can look at two relationships between rotational and translational motion.

1. Generally speaking, the linear kinematic equations have their rotational counterparts. Table 10.2 lists the four linear kinematic equations and the corresponding rotational counterpart. The two sets of equations look similar to each other, but describe two different physical situations, that is, rotation and translation.

Rotational	Translational
$\theta_f = \theta_0 + \bar{\omega}\, t$	$x = x_0 + \bar{v}\, t$
$\omega_f = \omega_0 + \alpha t$	$v_f = v_0 + at$
$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	$x_f = x_0 + v_0 t + \frac{1}{2}at^2$
$\omega_f^2 = \omega_0^2 + 2\alpha(\Delta\theta)$	$v_f^2 = v_0^2 + 2a(\Delta x)$

Table 10.2 Rotational and Translational Kinematic Equations

Example



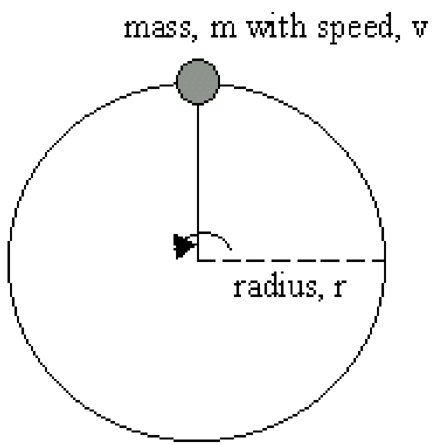
For the Brazos wind farm turbine, suppose it shuts down from 20 rpm to 0 rpm in a time span of one minute, with a constant angular acceleration.

What is the magnitude (in rad/s^2) and direction of α ? What is the tangential acceleration at the blade tip?

How many revolutions does the turbine make during the minute it takes to come to rest?

Rotational kinetic energy

Consider a single mass being swung in a circle on the end of a string with length r.



The kinetic energy is just

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m(r\omega)^2 = \frac{1}{2}(mr^2)\omega^2$$

Now if we have a large number of little masses connected together to form a hoop, then we add up all of the kinetic energies to get the total for the hoop.

$$K = \Sigma\frac{1}{2}m_j{v_j}^2 = \frac{1}{2}\Sigma m_j(r_j\omega)^2 = \frac{1}{2}\Sigma(m_j{r_j}^2)\omega^2$$

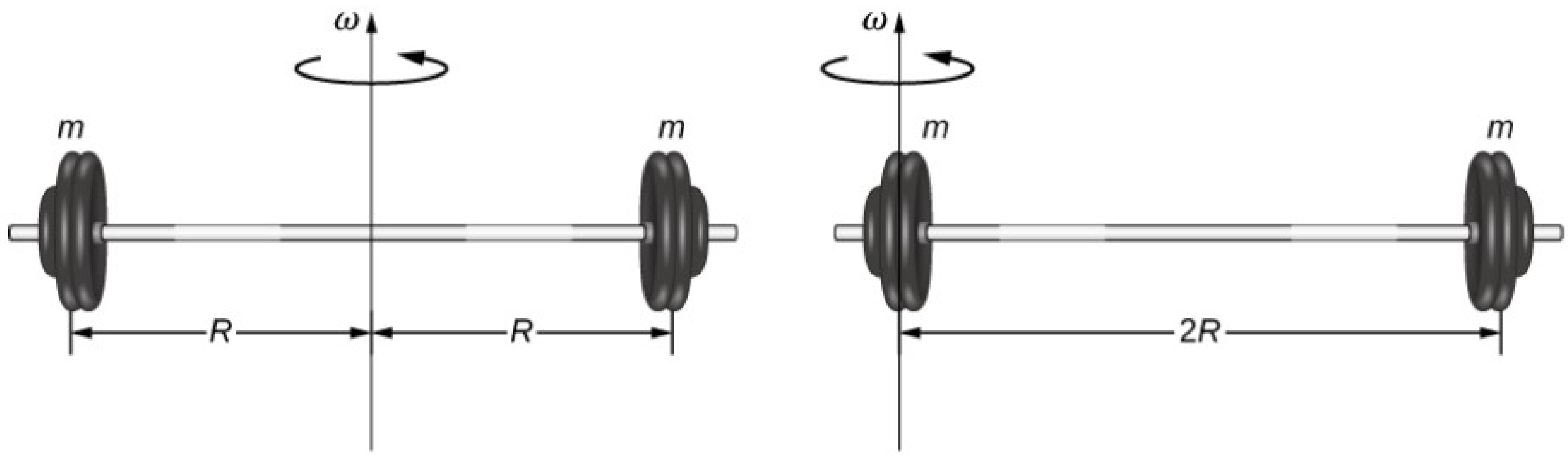
Note that for all solid rotating bodies, ω is the same at all points

Moment of Inertia for a distribution of discrete masses

for rotational motion. This quantity is called the **moment of inertia** *I*, with units of $\mathrm{kg \cdot m^2}$:

$$I = \sum_j m_j r_j^2. \qquad (10.17)$$

Example: for case (a), $I = mR^2 + mR^2 = 2mR^2$
for case (b), $I = m(0)^2 + m(2R)^2 = 4mR^2$



(a) (b)

The moment of inertia for an object depends on where we place the axis of rotation!

For objects with a continuous distribution of mass,

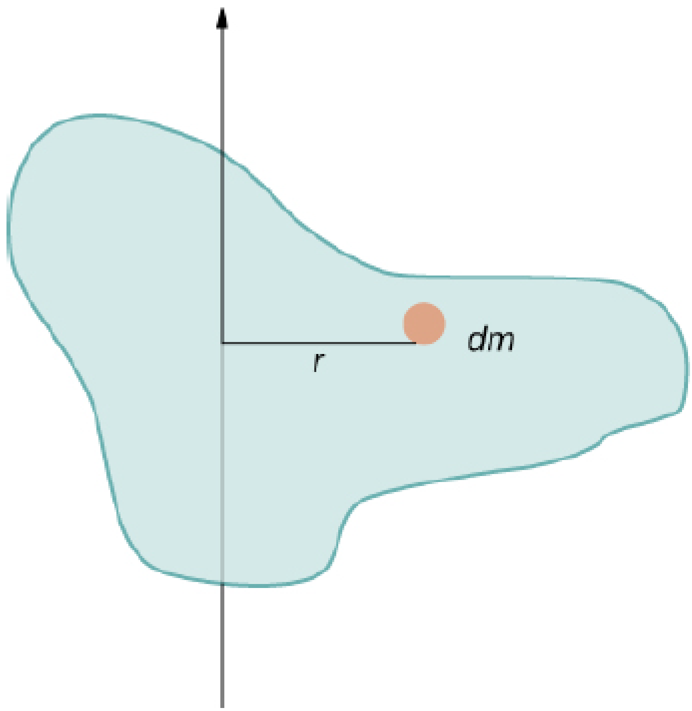
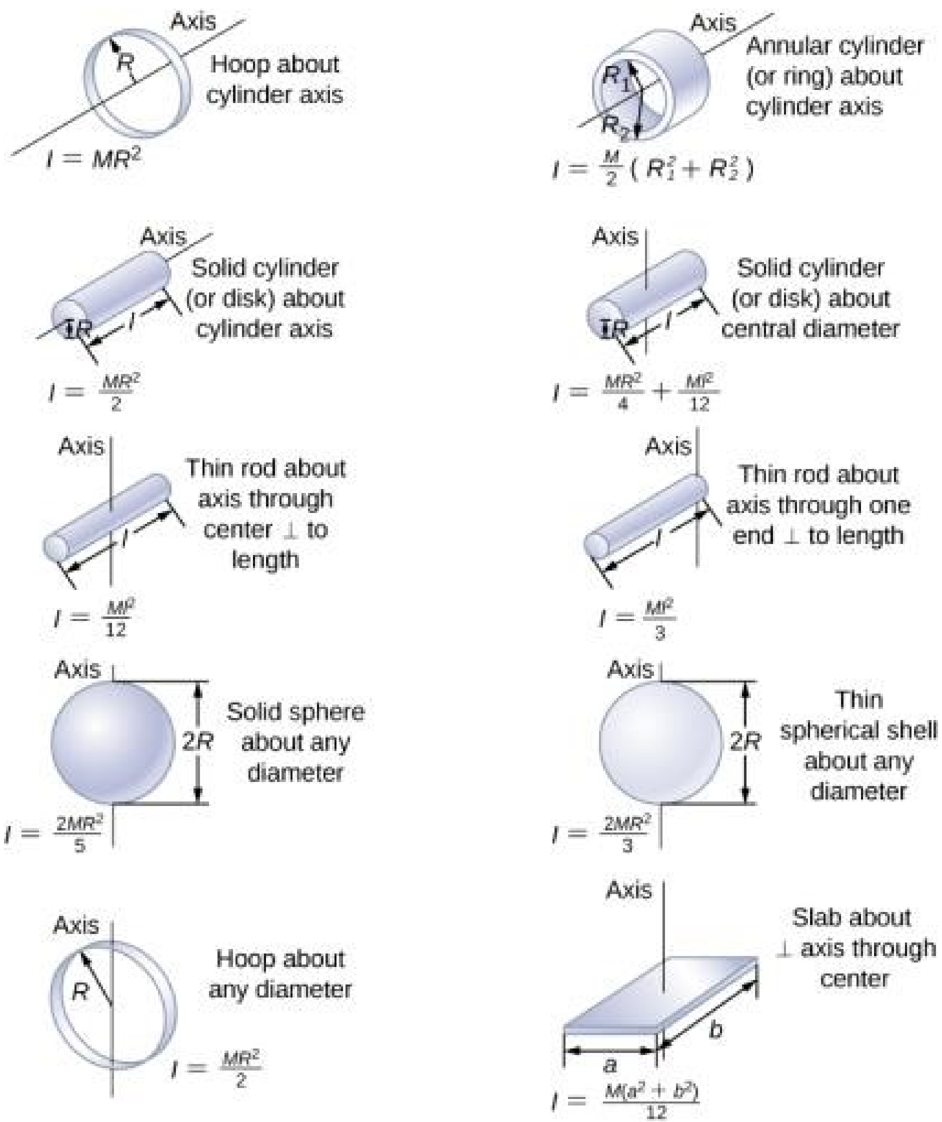


Figure 10.24 Using an infinitesimally small piece of mass to calculate the contribution to the total moment of inertia.

The need to use an infinitesimally small piece of mass *dm* suggests that we can write the moment of inertia by evaluating an integral over infinitesimal masses rather than doing a discrete sum over finite masses:

$$I = \sum_i m_i {r_i}^2 \text{ becomes } I = \int r^2 dm. \tag{10.19}$$

FIGURE 10.20

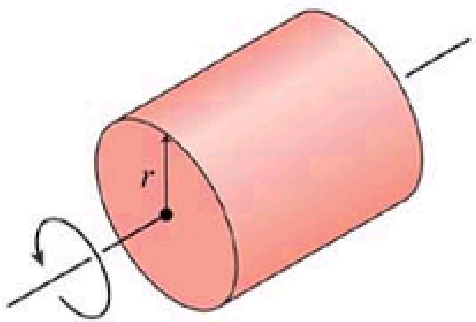


Values of rotational inertia for common shapes of objects

Note that the book goes through the calculations for various shapes and rotation axes, and presents the "parallel axis" theorem.

This is important material, but we will not be doing integral calculations or solving complicated mass distributions to find moments of inertia. For the most part, we can just refer to the results shown in this table.

Iclicker: A solid cylinder made of lead has the same mass and same length as a solid cylinder made of aluminum. The rotational inertia of the lead cylinder compared to the aluminum one is:



Disk or solid cylinder
about its axis
$I = \frac{1}{2}MR^2$

A. greater
B. less
C. same
D. unknown unless both radii are specified exactly

Now that we've defined the moment of inertia,

Rotational	Translational
$I = \sum_j m_j r_j^2$	m
$K = \frac{1}{2} I\omega^2$	$K = \frac{1}{2} mv^2$

Table 10.4 Rotational and Translational Kinetic Energies and Inertia

And the total energy becomes

$$E_T = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + U$$

Example:

(a) A hoop of mass M and radius R, and a solid disk of mass M and radius R, both **slide** down a frictionless ramp of height H, starting from rest. What are their velocities when they reach the bottom?

(b) Now suppose the hoop and disk both **roll** down the ramp. What are the final velocities?

Note: for rolling motion, it's easy to show (though not until Chap 11!) that the translational velocity is given by $v = r\omega$

ETA Problem 9.4.22

Professional Application *The Moon's craters are remnants of meteorite collisions. Suppose a fairly large asteroid that has a mass of* 5.00×10^{12} kg *(about a kilometer across) strikes the Moon at a speed of 15.0 km/s. (a) At what speed does the Moon recoil after the perfectly inelastic collision (the mass of the Moon is* 7.36×10^{22} kg*) ? (b) How much kinetic energy is lost in the collision? Such an event may have been observed by medieval English monks who reported observing a red glow and subsequent haze about the Moon. (c) In October 2009, NASA crashed a rocket into the Moon, and analyzed the plume produced by the impact. (Significant amounts of water were detected.) Answer part (a) and (b) for this real-life experiment. The mass of the rocket was 2000 kg and its speed upon impact was 9000 km/h. How does the plume produced alter these results?*