Physics 121 – November 28, 2017

Announcements:

- Final exam on Thursday, Dec 14 at 9:00 AM in Workman 101
- Review on Dec 7 (last class meeting)

Assignments:

- Finish reading Chapter 16.
- Complete ETA Problem Set #15 (last one!) by Monday, Dec 4.
- End-of-chapter problems: Ch 16: 70, 71, 81, 87, 98, 102, and 114. Due by 4 pm, Dec 4.
- Recitation practice problems 69, 82, 97, 103, and 106

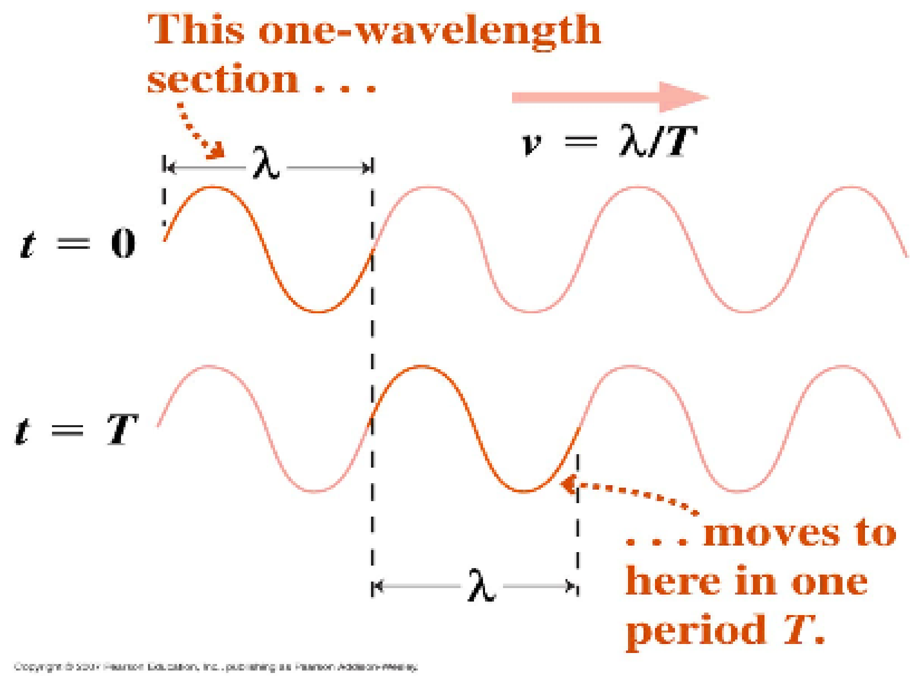
Tips for Chapter 16 HW

- #81: Assume that the ocean wave velocity and wavelength remain constant.
- #87: Change the clause, "the intensity ~~at the source~~ is I_1 at a distance of one meter from the source" .
- #114: You want to find the smallest overtone frequency that is greater than 100 Hz.

Quick Review:

Traveling waves:

v = wave speed
T = period
ω = angular frequency
$f = \omega/(2\pi)$ = frequency
λ = wavelength
$k = 2\pi/\lambda$ = angular wavenumber



Transverse wave: motions of mass element are *perpendicular* to the direction of wave motion.
Longitudinal wave: motions of mass element are *parallel* to the direction of wave motion.

$$v = \frac{\lambda}{T} = \frac{\lambda}{T}\left(\frac{2\pi}{2\pi}\right) = \frac{\omega}{k}. \quad (16.3)$$

Think back to our discussion of a mass on a spring, when the position of the mass was modeled as $x(t) = A\cos(\omega t + \phi)$. The angle ϕ is a phase shift, added to allow for the fact that the mass may have initial conditions other than $x = +A$ and $v = 0$. For similar reasons, the initial phase is added to the wave function. The wave function modeling a sinusoidal wave, allowing for an initial phase shift ϕ, is

$$y(x,\, t) = A\sin(kx \mp \omega t + \phi) \quad (16.4)$$

The value

$$(kx \mp \omega t + \phi) \quad (16.5)$$

is known as the phase of the wave, where ϕ is the initial phase of the wave function. Whether the temporal term ωt is negative or positive depends on the direction of the wave. First consider the minus sign for a wave with an initial phase equal to zero $(\phi = 0)$. The phase of the wave would be $(kx - \omega t)$. Consider following a point on a wave, such as a crest. A crest will occur when $\sin(kx - \omega t) = 1.00$, that is, when $kx - \omega t = n\pi + \frac{\pi}{2}$, for any integral value of *n*. For instance, one particular crest occurs at $kx - \omega t = \frac{\pi}{2}$. As the wave moves, time increases and *x* must also increase to keep the phase equal to $\frac{\pi}{2}$. Therefore, the minus sign is for a wave moving in the positive *x*-direction. Using the plus sign, $kx + \omega t = \frac{\pi}{2}$. As time increases, *x* must decrease to keep the phase equal to $\frac{\pi}{2}$. The plus sign is used for waves moving in the negative *x*-direction. In summary, $y(x,\, t) = A\sin(kx - \omega t + \phi)$ models a wave moving in the positive *x*-direction and $y(x,\, t) = A\sin(kx + \omega t + \phi)$ models a wave moving in the negative *x*-direction.

Energy and Power in Waves:

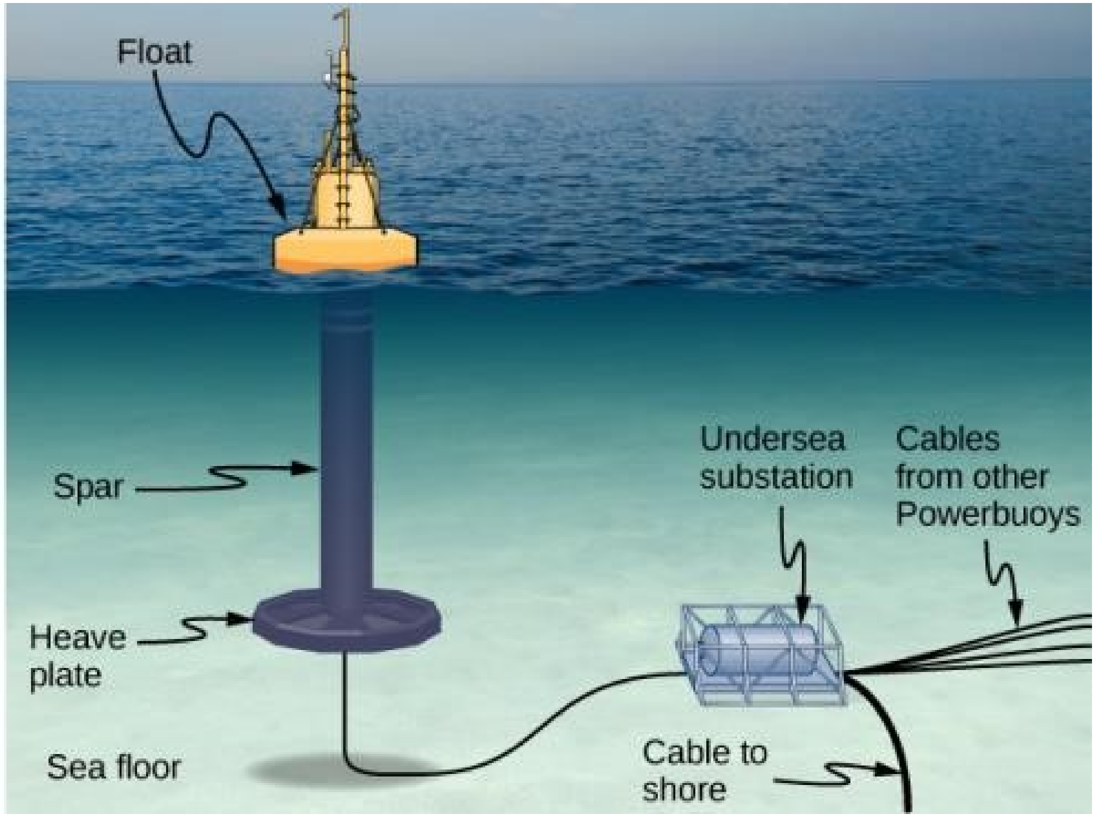
$$E_\lambda = U_\lambda + K_\lambda,$$
$$E_\lambda = \frac{1}{4}\mu A^2 \omega^2 \lambda + \frac{1}{4}\mu A^2 \omega^2 \lambda = \frac{1}{2}\mu A^2 \omega^2 \lambda.$$

The time-averaged power of a sinusoidal mechanical wave, which is the average rate of energy transfer associated with a wave as it passes a point, can be found by taking the total energy associated with the wave divided by the time it takes to transfer the energy. If the velocity of the sinusoidal wave is constant, the time for one wavelength to pass by a point is equal to the period of the wave, which is also constant. For a sinusoidal mechanical wave, the time-averaged power is therefore the energy associated with a wavelength divided by the period of the wave. The wavelength of the wave divided by the period is equal to the velocity of the wave,

$$P_{\text{ave}} = \frac{E_\lambda}{T} = \frac{1}{2}\mu A^2 \omega^2 \frac{\lambda}{T} = \frac{1}{2}\mu A^2 \omega^2 v. \quad (16.10)$$

Note that this equation for the time-averaged power of a sinusoidal mechanical wave shows that the power is proportional to the square of the amplitude of the wave and to the square of the angular frequency of the wave. Recall that the angular frequency is equal to $\omega = 2\pi f$, so the power of a mechanical wave is equal to the square of the amplitude and the square of the frequency of the wave.

FIGURE 16.1



From the world of renewable energy sources comes the electric power-generating buoy. Although there are many versions, this one converts the up-and-down motion, as well as side-to-side motion, of the buoy into rotational motion in order to turn an electric generator, which stores the energy in batteries.

Power from this buoy should be proportional to the wave amplitude squared

A related concept is the Intensity carried by waves (ETA problem 1, Chapter 16 problem 87.

Another important characteristic of waves is the intensity of the waves. Waves can also be concentrated or spread out. Waves from an earthquake, for example, spread out over a larger area as they move away from a source, so they do less damage the farther they get from the source. Changing the area the waves cover has important effects. All these pertinent factors are included in the definition of **intensity (I)** as power per unit area:

$$I = \frac{P}{A}, \tag{16.11}$$

where P is the power carried by the wave through area A. The definition of intensity is valid for any energy in transit, including that carried by waves. The SI unit for intensity is watts per square meter (W/m^2). Many waves are spherical waves that move out from a source as a sphere. For example, a sound speaker mounted on a post above the ground may produce sound waves that move away from the source as a spherical wave. Sound waves are discussed in more detail in the next chapter, but in general, the farther you are from the speaker, the less intense the sound you hear. As a spherical wave moves out from a source, the surface area of the wave increases as the radius increases $\left(A = 4\pi r^2\right)$. The intensity for a spherical wave is therefore

$$I = \frac{P}{4\pi r^2}. \tag{16.12}$$

Let's look at a few examples:

1. An omnidirectional speaker
2. A small laser pointer

Consider an omnidirectional speaker driven by a 200-Watt audio amplifier. Typically, only 1% of the electrical energy from the amplifier is converted to acoustic energy (the rest is mostly dissipated as heat).

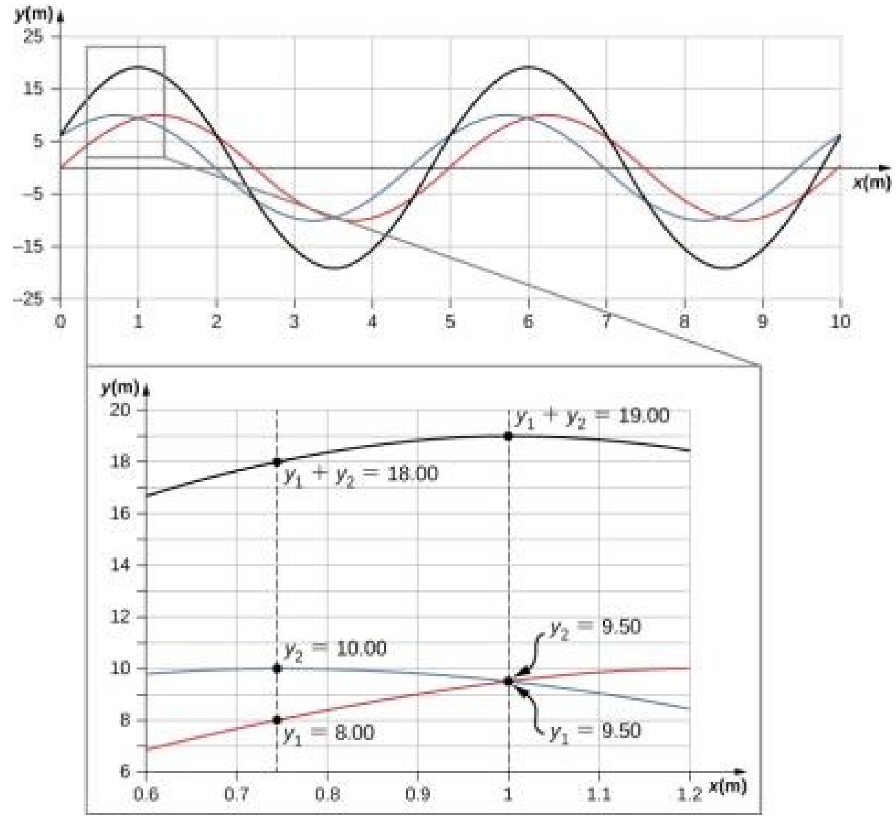
What is the acoustic wave intensity 4 meters from the speaker, assuming a spherical sound wave?

Consider an omnidirectional speaker driven by a 200-Watt audio amplifier. Typically, only 1% of the electrical energy from the amplifier is converted to acoustic energy (the rest is mostly dissipated as heat). We found 100 dB level at 4 m from speaker.

Sound intensity level β (dB)	**Intensity I $(\mathrm{W/m^2})$**	**Example/effect**
0	1×10^{-12}	Threshold of hearing at 1000 Hz
10	1×10^{-11}	Rustle of leaves
20	1×10^{-10}	Whisper at 1-m distance
30	1×10^{-9}	Quiet home
40	1×10^{-8}	Average home
50	1×10^{-7}	Average office, soft music
60	1×10^{-6}	Normal conversation
70	1×10^{-5}	Noisy office, busy traffic
80	1×10^{-4}	Loud radio, classroom lecture
90	1×10^{-3}	Inside a heavy truck; damage from prolonged exposure[1]
100	1×10^{-2}	Noisy factory, siren at 30 m; damage from 8 h per day exposure
110	1×10^{-1}	Damage from 30 min per day exposure
120	1	Loud rock concert; pneumatic chipper at 2 m; threshold of pain
140	1×10^{2}	Jet airplane at 30 m; severe pain, damage in seconds
160	1×10^{4}	Bursting of eardrums

Superposition of waves:

FIGURE 16.22



When two linear waves in the same medium interfere, the height of resulting wave is the sum of the heights of the individual waves, taken point by point. This plot shows two waves (red and blue) added together, along with the resulting wave (black). These graphs represent the height of the wave at each point. The waves may be any linear wave, including ripples on a pond, disturbances on a string, sound, or electromagnetic waves.

FIGURE 16.24

Superposition of two waves with identical amplitudes, wavelengths, and frequency, but that differ in a phase shift. The red wave is defined by the wave function $y_1(x, t) = A \sin(kx - \omega t)$ and the blue wave is defined by the wave function $y_2(x, t) = A \sin(kx - \omega t + \phi)$. The black line shows the result of adding the two waves. The phase difference between the two waves are (a) 0.00 rad, (b) $\pi/2$ rad, (c) π rad, and (d) $3\,\pi/2$ rad.

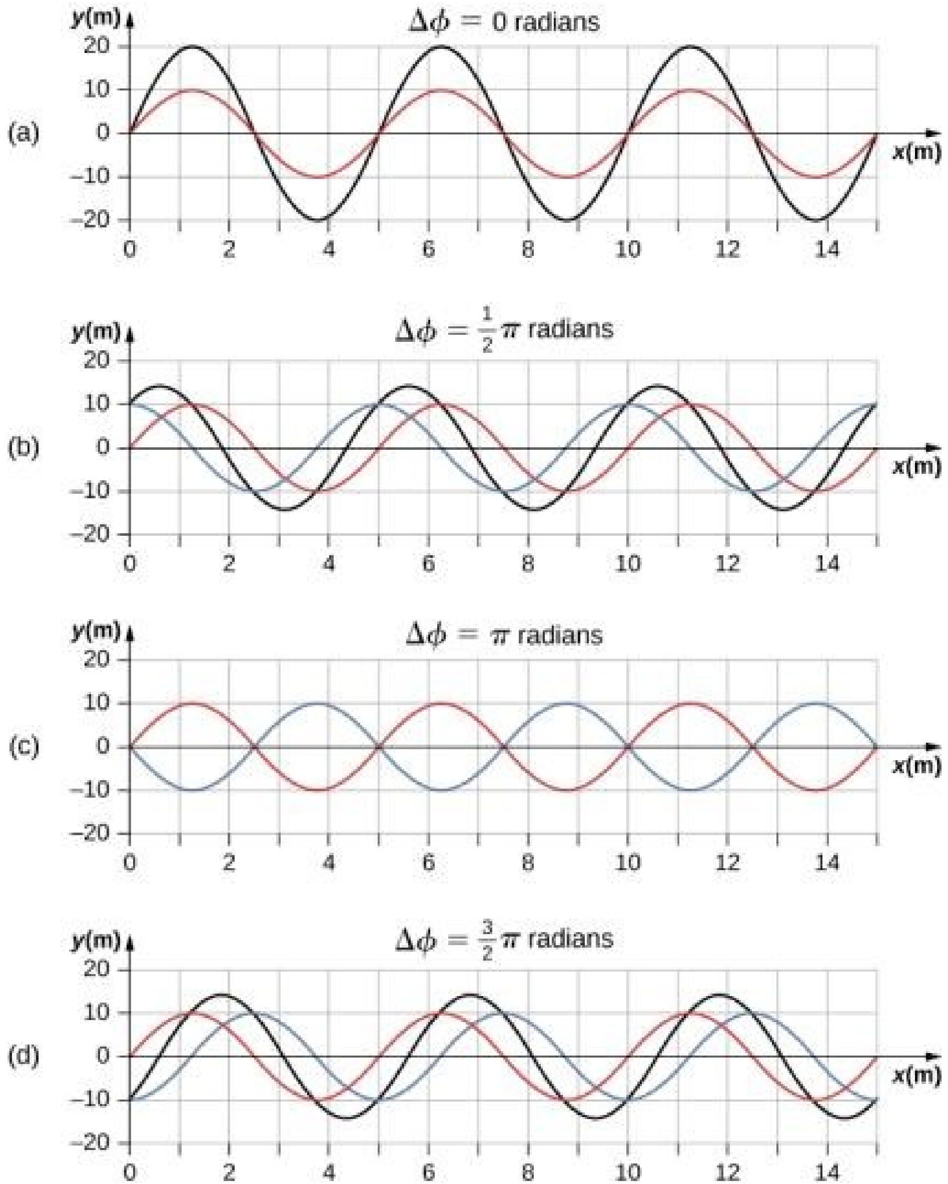
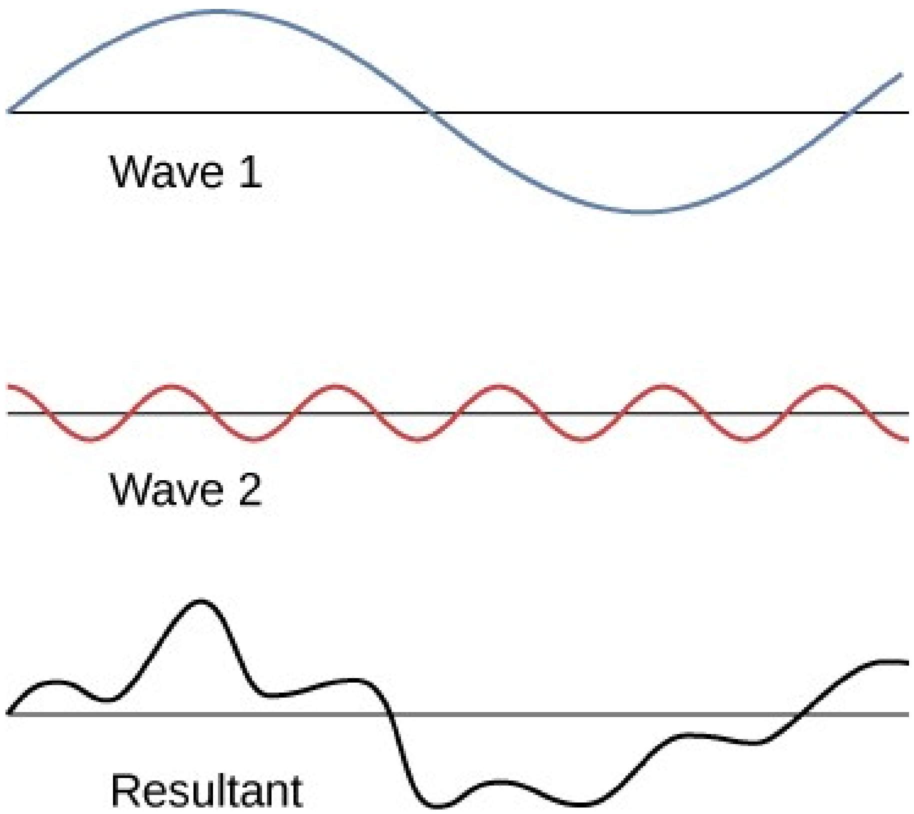


FIGURE 16.23



Superposition of nonidentical waves exhibits both constructive and destructive interference.

You *may* learn about Fourier decomposition and analysis in a math course.

Quick Review:

Traveling waves on a string

A critically important quantity is the linear mass density (mass per unit length)

$$\mu = \frac{\text{mass of string}}{\text{length of string}} = \frac{m}{l}. \tag{16.7}$$

In this chapter, we consider only string with a constant linear density. If the linear density is constant, then the mass (Δm) of a small length of string (Δx) is $\Delta m = \mu \Delta x$. For example, if the string has a length of 2.00 m and a mass of 0.06 kg, then the linear density is $\mu = \frac{0.06\text{ kg}}{2.00\text{ m}} = 0.03\frac{\text{kg}}{\text{m}}$. If a 1.00-mm section is cut from the string, the mass of the 1.00-mm length is $\Delta m = \mu \Delta x = \left(0.03\frac{\text{kg}}{\text{m}}\right)0.001\text{ m} = 3.00 \times 10^{-5}\text{ kg}$. The guitar also has a method to change the tension of the strings. The tension of the strings is adjusted by turning spindles, called the tuning pegs, around which the strings are wrapped. For the guitar, the linear density of the string and the tension in the string determine the speed of the waves in the string and the frequency of the sound produced is proportional to the wave speed.

Wave Speed on a String under Tension

To see how the speed of a wave on a string depends on the tension and the linear density, consider a pulse sent down a taut string (Figure 16.13). When the taut string is at rest at the equilibrium position, the tension in the string F_T is constant. Consider a small element of the string with a mass equal to $\Delta m = \mu \Delta x$. The mass element is at rest and in equilibrium and the force of tension of either side of the mass element is equal and opposite.

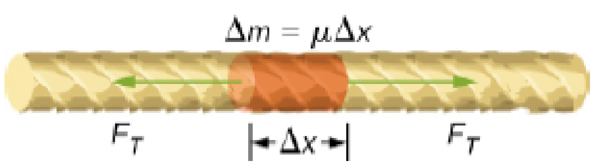


Figure 16.13 Mass element of a string kept taut with a tension F_T. The mass element is in static equilibrium, and the force of tension acting on either side of the mass element is equal in magnitude and opposite in direction.

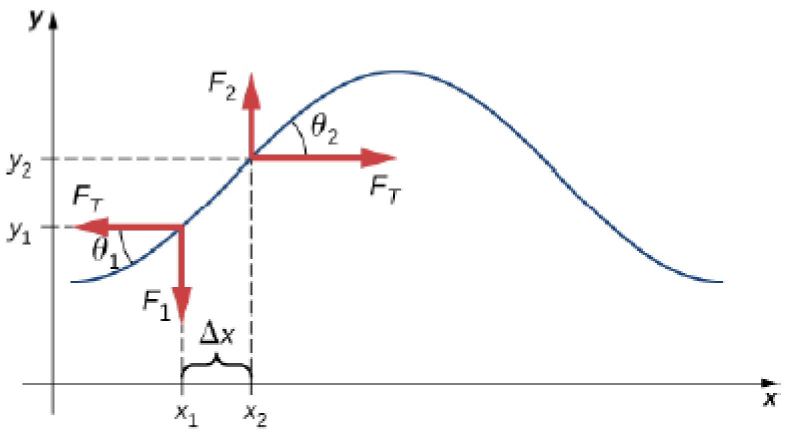


Figure 16.14 A string under tension is plucked, causing a pulse to move along the string in the positive *x*-direction.

The derivation of the wave equation for a string under tension (pp 824-825) is rather complicated, but we can qualitatively explain the result.

$$\frac{\partial^2 y(x,\, t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x,\, t)}{\partial t^2}.$$

Therefore,

$$\frac{1}{v^2} = \frac{\mu}{F_T}.$$

Solving for v, we see that the speed of the wave on a string depends on the tension and the linear density.

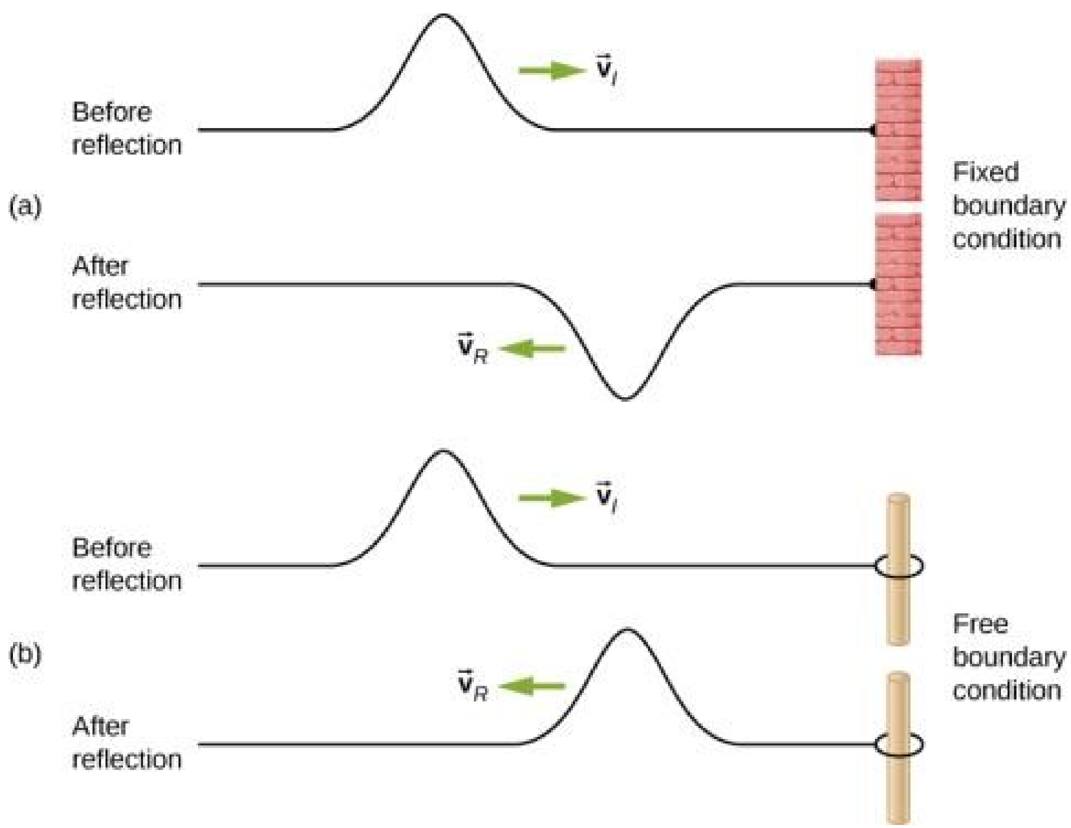
Speed of a Wave on a String Under Tension

The speed of a pulse or wave on a string under tension can be found with the equation

$$|v| = \sqrt{\frac{F_T}{\mu}} \tag{16.8}$$

where F_T is the tension in the string and μ is the mass per length of the string.

Figure 16.17

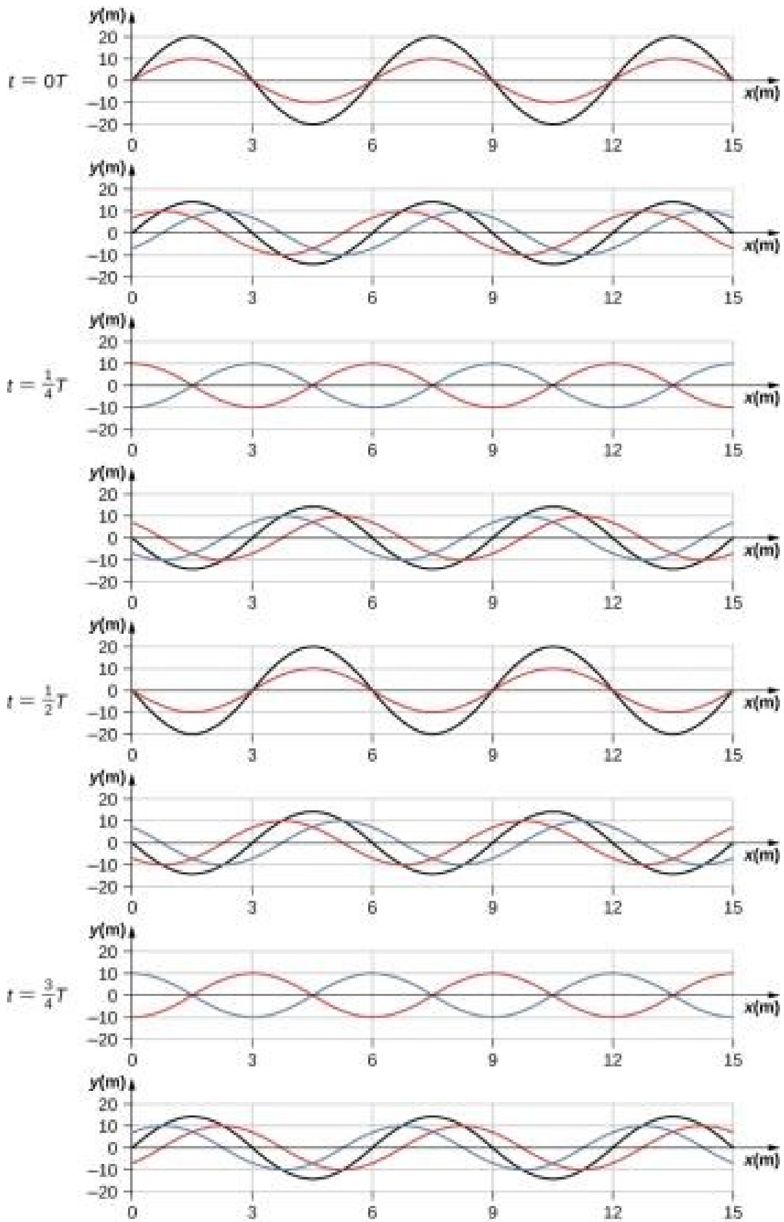


(a) One end of a string is fixed so that it cannot move. A wave propagating on the string, encountering this fixed boundary condition, is reflected 180˚(π rad) out of phase with respect to the incident wave.

(b) One end of a string is tied to a solid ring of negligible mass on a frictionless lab pole, where the ring is free to move. A wave propagating on the string, encountering this free boundary condition, is reflected in phase 0˚ (0 rad) with respect to the wave.

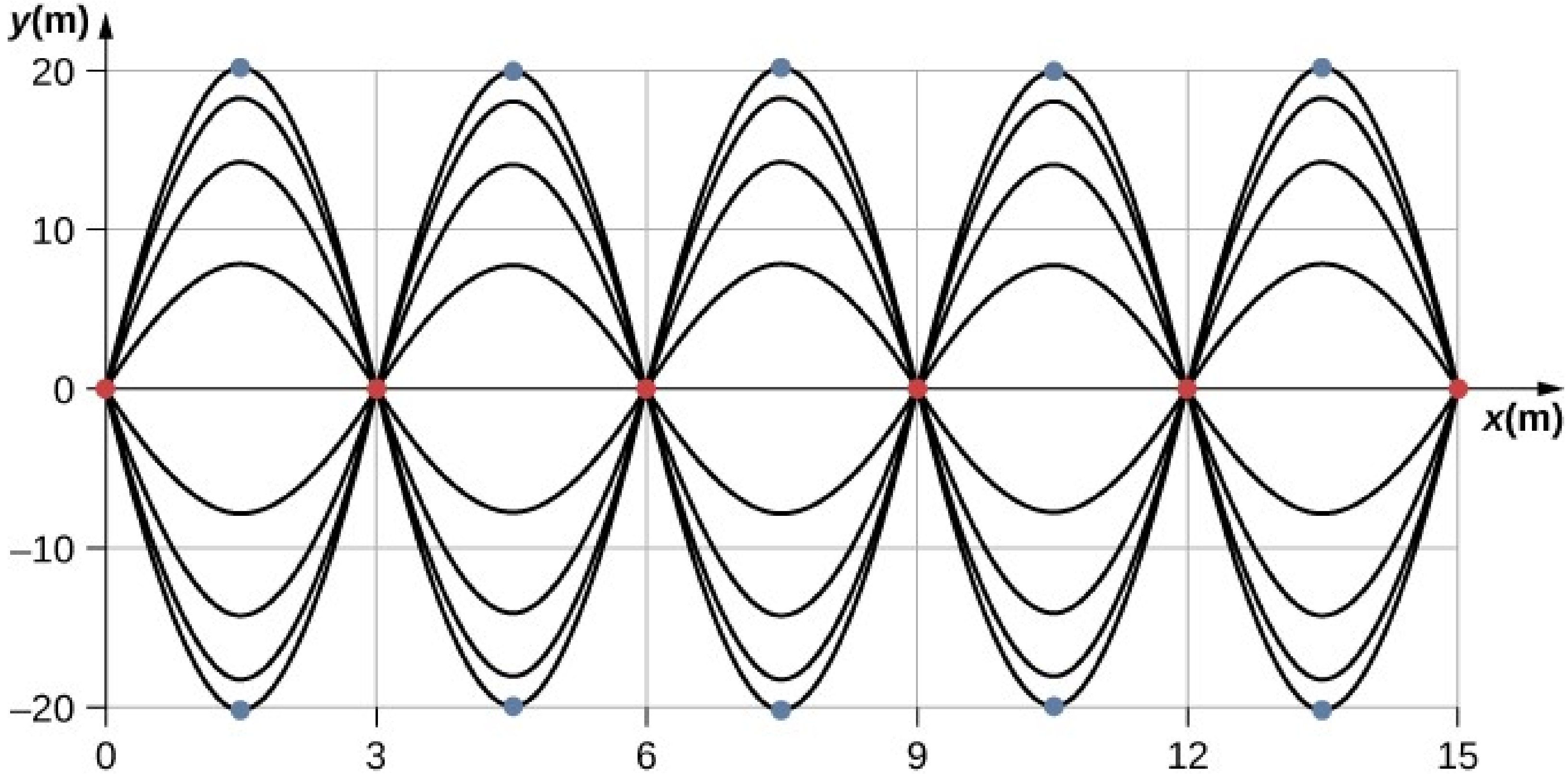
Check out this [wave simulation]

Figure 16.26



Time snapshots of two sine waves. The red wave is moving in the $-x$-direction and the blue wave is moving in the $+x$-direction. The resulting wave is shown in black. Consider the resultant wave at the points $x = 0$ m, 3 m, 6 m, 9 m, 12 m, 15 m and notice that the resultant wave always equals zero at these points, no matter what the time is. These points are known as fixed points (nodes). In between each two nodes is an antinode, a place where the medium oscillates with an amplitude equal to the sum of the amplitudes of the individual waves.

Figure 16.27



When two identical waves are moving in opposite directions, the resultant wave is a standing wave. Nodes appear at integer multiples of half wavelengths. Antinodes appear at odd multiples of quarter wavelengths, where they oscillate between $y = \pm A$. The nodes are marked with red dots and the antinodes are marked with blue dots.