Physics 121 – November 21, 2017

Announcements:

No labs or recitations this week

Assignments:

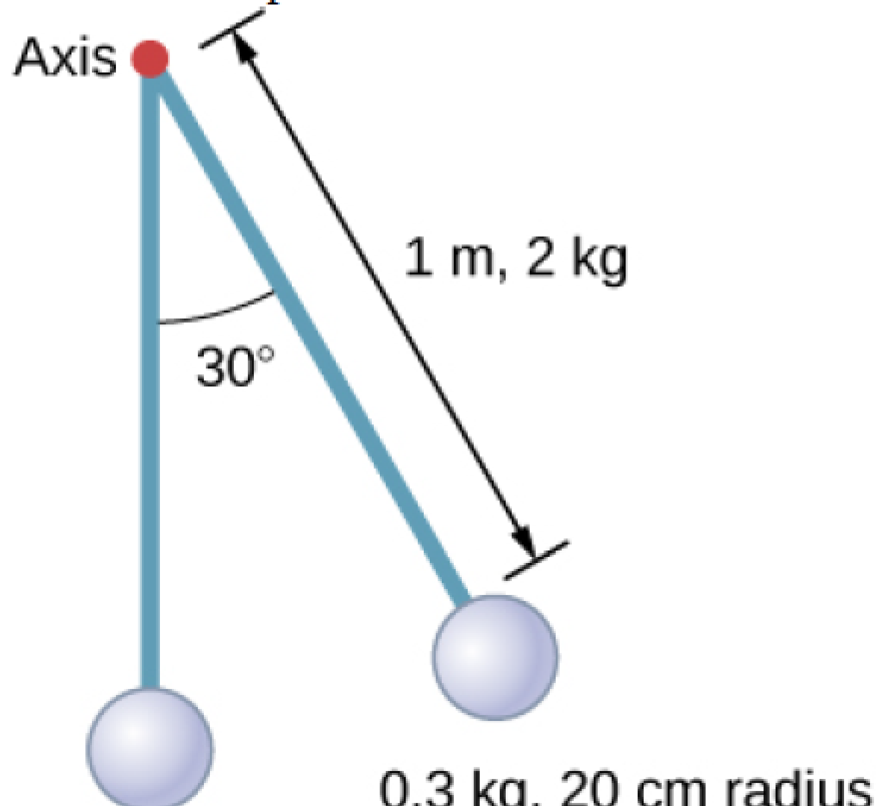
This week:

- Read Chapter 16.
- Complete ETA Problem Set #14 by Monday, Nov 27.
- End-of-chapter problems: Ch 16: 40, 42, 48, and 55.

Due by 4 pm, Nov 27.

Let's take a quick look at a HW problem from Chapter 10

68. A pendulum consists of a rod of mass 2 kg and length 1 m with a solid sphere at one end with mass 0.3 kg and radius 20 cm (see the following figure). If the pendulum is released from rest at an angle of $30°$, what is the angular velocity at the lowest point?



1. Need to use conservation of energy $\Delta K = \Delta U$
2. First calculate center of mass to get gravitational $\Delta U = mg\Delta h_{cm}$
3. To find ΔK, need the moment of inertia of the rod-sphere system
4. Solve for ω

Solution

the center of mass of the system is located at

$$x_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{2\text{ kg}(0.5\text{ m}) + 0.3\text{ kg}(1.2\text{ m})}{(2.0+0.3)\text{kg}} = 0.6\text{ m} \text{ from the axis of rotation.}$$

$$I_{\text{pend}} = \frac{1}{3}(2\text{ kg})(1.0\text{ m})^2 + \frac{2}{5}(0.3\text{ kg})(0.2\text{ m})^2 + 0.3\text{ kg}(1.2\text{ m})^2 = 1.1\text{ kg}\cdot\text{m}^2,$$

$$\Delta U = mg\Delta h = (2.3\text{ kg})(9.8)0.6\text{ m}(1-\cos 30) = 1.8\text{ J}, \text{ and}$$

$$\Delta K = \frac{1}{2}I\omega^2 = \frac{1}{2}(1.1\text{ kg}\cdot\text{m}^2)\omega^2 \Rightarrow \omega = \sqrt{\frac{1.8\text{ J}}{0.55\text{ kg}\cdot\text{m}^2}} = 1.82\text{ rad/s}$$

Note – given the scale in the problem, you could just treat the sphere as a point mass located 1.2 m from the pivot (neglecting the second term in I_{pend}, and just keeping the third term).

Now look at problem 51 from Chapter 15 (and last ETA problem)

The amplitude of a lightly damped harmonic oscillator decreases by 3.0% during each cycle.

a. What percentage of the mechanical energy of the oscillator is lost in each cycle?

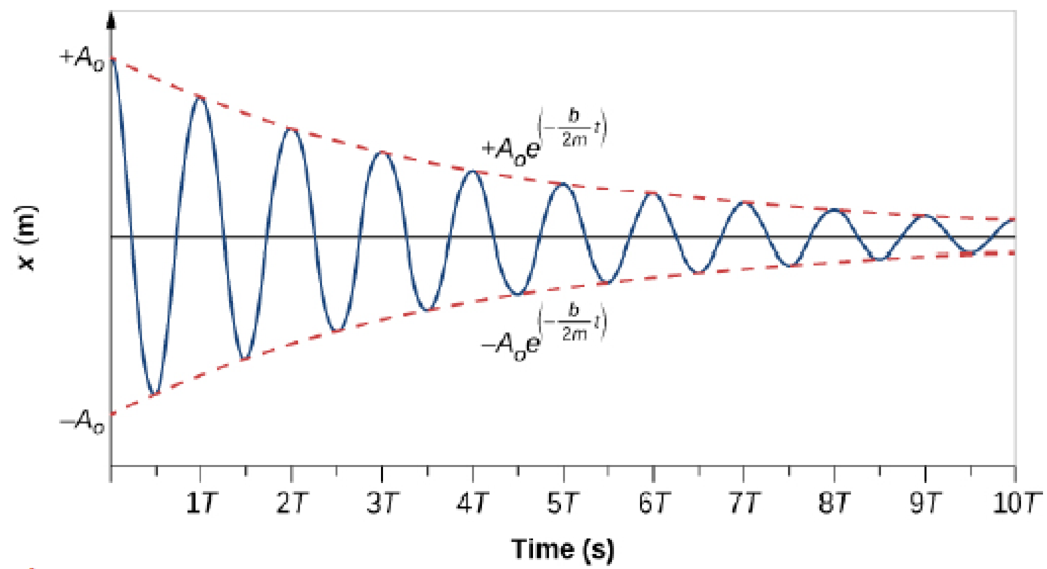
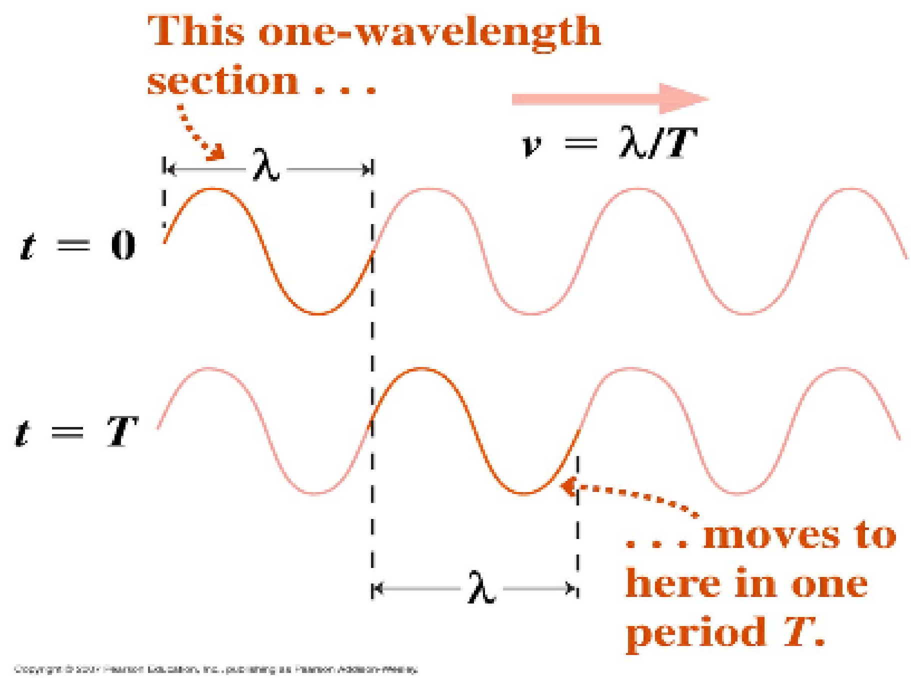


Figure 15.26 Position versus time for the mass oscillating on a spring in a viscous fluid. Notice that the curve appears to be a cosine function inside an exponential envelope.

Answer in back of book is wrong!
Also, what is meant by "lightly damped"?
The correct term is "underdamped".

Traveling waves:



v = wave speed

T = period

ω = angular frequency

$f = \omega/(2\pi)$ = frequency

λ = wavelength

$k = 2\pi/\lambda$ = angular wavenumber

Transverse wave: motions of mass element are *perpendicular* to the direction of wave motion.

Longitudinal wave: motions of mass element are *parallel* to the direction of wave motion.

Pretty cool simulation for traveling [longitudinal and tranverse waves]()

iclicker:

Consider a traveling wave on a string shown below at some instant in time which we define as t=0 The direction of the instantaneous velocity of a small piece of the string located at point P, at time t=0, is

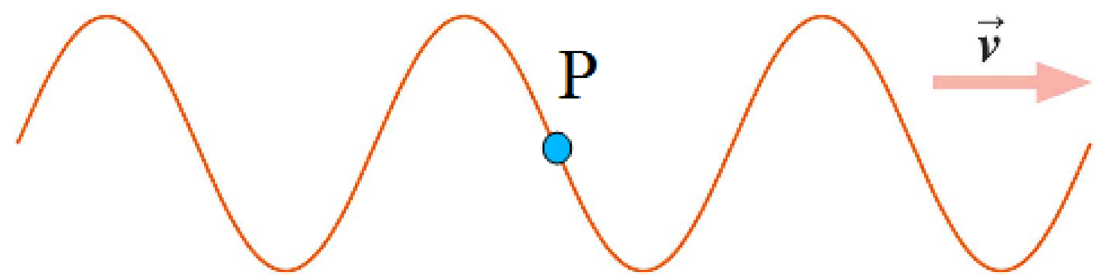
A ↓

B ↑

C ↗

D ↙

E →



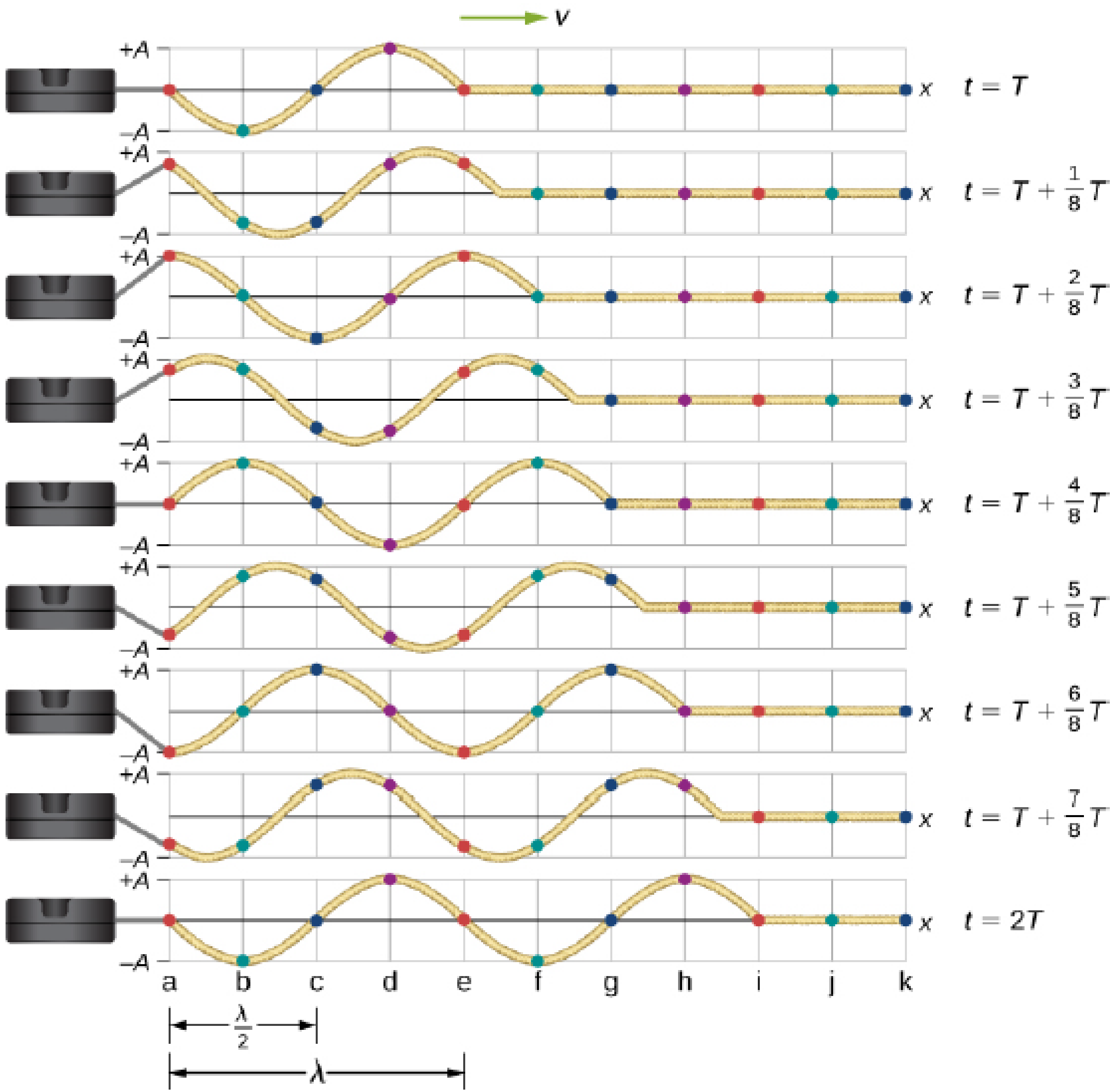


Figure 16.9 Snapshots of a transverse wave moving through a string under tension, beginning at time $t = T$ and taken at intervals of $\frac{1}{8}T$. Colored dots are used to highlight points on the string.

On the other hand, we just saw that the motion on the surface of water waves is *circular*, not up-and-down as depicted here.

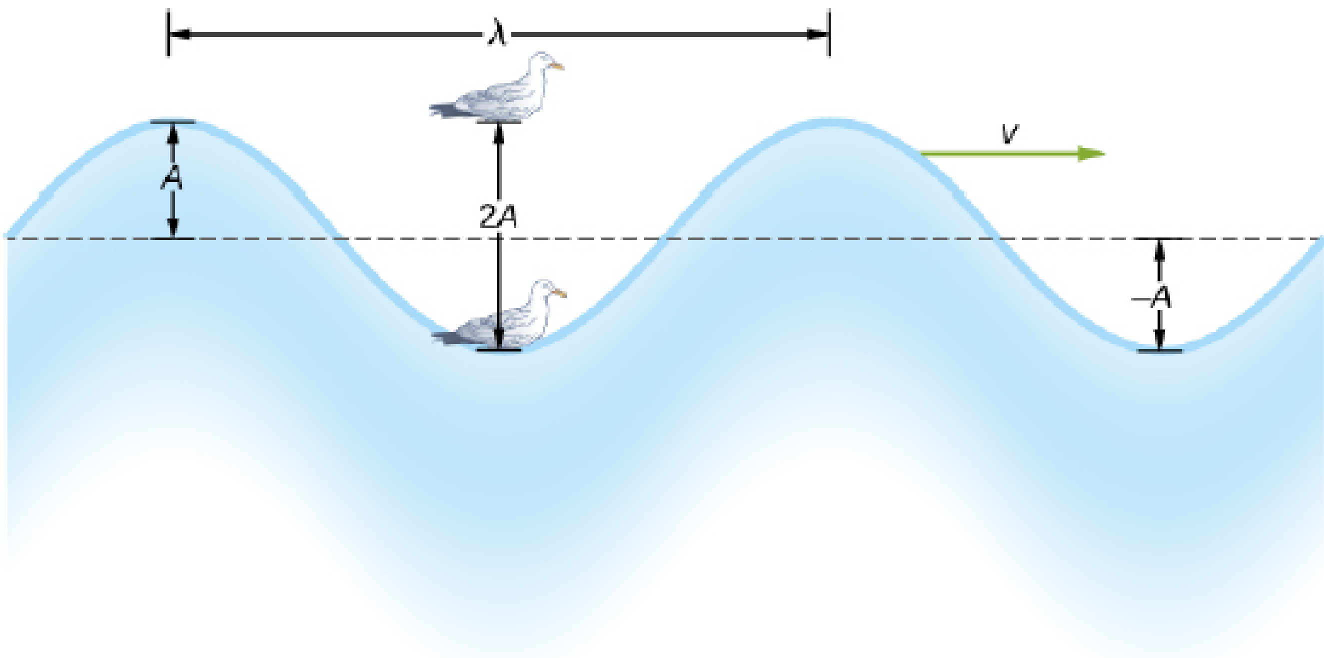


Figure 16.3 An idealized surface water wave passes under a seagull that bobs up and down in simple harmonic motion. The wave has a wavelength λ, which is the distance between adjacent identical parts of the wave. The amplitude A of the wave is the maximum displacement of the wave from the equilibrium position, which is indicated by the dotted line. In this example, the medium moves up and down, whereas the disturbance of the surface propagates parallel to the surface at a speed v.

This is an idealized situation that allows us to use all of the tools we develop for waves on strings.

ETA Problem 16.1.2

Note that the wavelength of this note in air ($v \sim 340$ m/s) is about ¼ the wavelength in water (also, HW #42).

$$v = \frac{\lambda}{T} = \frac{\lambda}{T}\left(\frac{2\pi}{2\pi}\right) = \frac{\omega}{k}. \tag{16.3}$$

Think back to our discussion of a mass on a spring, when the position of the mass was modeled as $x(t) = A\cos(\omega t + \phi)$. The angle ϕ is a phase shift, added to allow for the fact that the mass may have initial conditions other than $x = +A$ and $v = 0$. For similar reasons, the initial phase is added to the wave function. The wave function modeling a sinusoidal wave, allowing for an initial phase shift ϕ, is

$$y(x,\, t) = A\,\sin(kx \mp \omega t + \phi) \tag{16.4}$$

The value

$$(kx \mp \omega t + \phi) \tag{16.5}$$

is known as the phase of the wave, where ϕ is the initial phase of the wave function. Whether the temporal term ωt is negative or positive depends on the direction of the wave. First consider the minus sign for a wave with an initial phase equal to zero $(\phi = 0)$. The phase of the wave would be $(kx - \omega t)$. Consider following a point on a wave, such as a crest. A crest will occur when $\sin(kx - \omega t) = 1.00$, that is, when $kx - \omega t = n\pi + \frac{\pi}{2}$, for any integral value of *n*. For instance, one particular crest occurs at $kx - \omega t = \frac{\pi}{2}$. As the wave moves, time increases and *x* must also increase to keep the phase equal to $\frac{\pi}{2}$. Therefore, the minus sign is for a wave moving in the positive *x*-direction. Using the plus sign, $kx + \omega t = \frac{\pi}{2}$. As time increases, *x* must decrease to keep the phase equal to $\frac{\pi}{2}$. The plus sign is used for waves moving in the negative *x*-direction. In summary, $y(x,\, t) = A\,\sin(kx - \omega t + \phi)$ models a wave moving in the positive *x*-direction and $y(x,\, t) = A\,\sin(kx + \omega t + \phi)$ models a wave moving in the negative *x*-direction.

iclicker:

A wave is described by the equation

$$y(x,t) = 0.1\sin(3x + 10t)$$

where x is in meters, y is in centimeters, and t is in seconds. The wavelength is

A. 6π m

B. 3π m

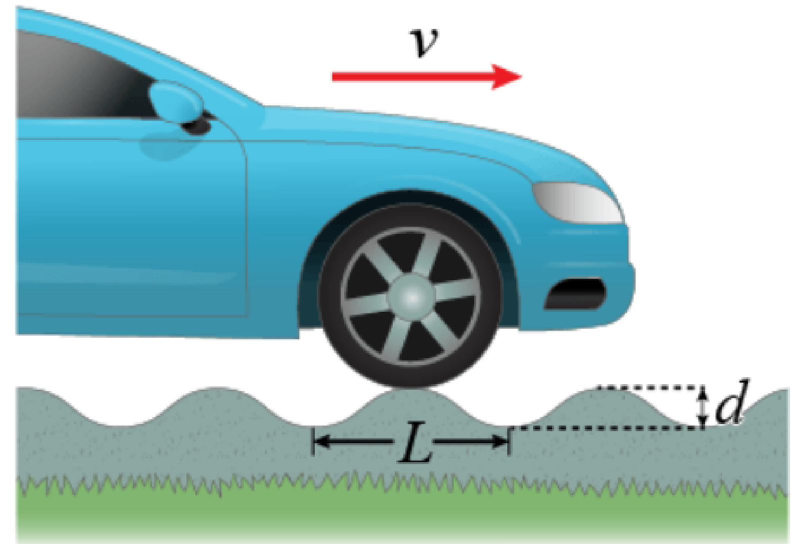
C. $2\pi/3$ m

D. $\pi/3$ m

E. 0.1 cm

ETA Problem Set 14 Begin Date: 11/20/2017 4:00:00 PM -- **Due Date:** 11/27/2017 4:00:00 PM **End Date:** 12/8/2017 11:00:00 PM

(20%) Problem 5: Special sections of roadway are sometimes paved with "rumble strips" to alert inattentive drivers. In a particular case the grooves are spaced L = *0.26* m apart and the depth of each groove is d = *0.35* cm. As you drive over this rumble strip, the tires of your car oscillate about their equilibrium positions with a frequency of f = *52* Hz. Refer to the figure, which is not drawn to scale.



33% Part (a) Enter an expression that describes the vertical position, $y(t)$, of one of the car tires as a function of time, t, in terms of the defined quantities. Assume the motion is sinusoidal, with its argument in radians and the positive y-axis up. Take the tire's equilibrium position as $y = 0$ and take $y(0) = 0$ and increasing.

33% Part (b) Find the vertical position of the tire, in centimeters, at the time t = *2.6* s.

For part (a), be sure to pay attention to the last sentence.
For part (b), remember to use radians in your trig function!

Traveling waves on a string

A critically important quantity is the linear mass density (mass per unit length)

$$\mu = \frac{\text{mass of string}}{\text{length of string}} = \frac{m}{l}. \tag{16.7}$$

In this chapter, we consider only string with a constant linear density. If the linear density is constant, then the mass (Δm) of a small length of string (Δx) is $\Delta m = \mu \Delta x$. For example, if the string has a length of 2.00 m and a mass of 0.06 kg, then the linear density is $\mu = \frac{0.06\text{ kg}}{2.00\text{ m}} = 0.03\frac{\text{kg}}{\text{m}}$. If a 1.00-mm section is cut from the string, the mass of the 1.00-mm length is $\Delta m = \mu \Delta x = \left(0.03\frac{\text{kg}}{\text{m}}\right)0.001\text{ m} = 3.00 \times 10^{-5}\text{ kg}$. The guitar also has a method to change the tension of the strings. The tension of the strings is adjusted by turning spindles, called the tuning pegs, around which the strings are wrapped. For the guitar, the linear density of the string and the tension in the string determine the speed of the waves in the string and the frequency of the sound produced is proportional to the wave speed.

Wave Speed on a String under Tension

To see how the speed of a wave on a string depends on the tension and the linear density, consider a pulse sent down a taut string (Figure 16.13). When the taut string is at rest at the equilibrium position, the tension in the string F_T is constant. Consider a small element of the string with a mass equal to $\Delta m = \mu \Delta x$. The mass element is at rest and in equilibrium and the force of tension of either side of the mass element is equal and opposite.

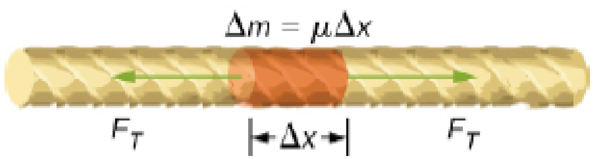


Figure 16.13 Mass element of a string kept taut with a tension F_T. The mass element is in static equilibrium, and the force of tension acting on either side of the mass element is equal in magnitude and opposite in direction.

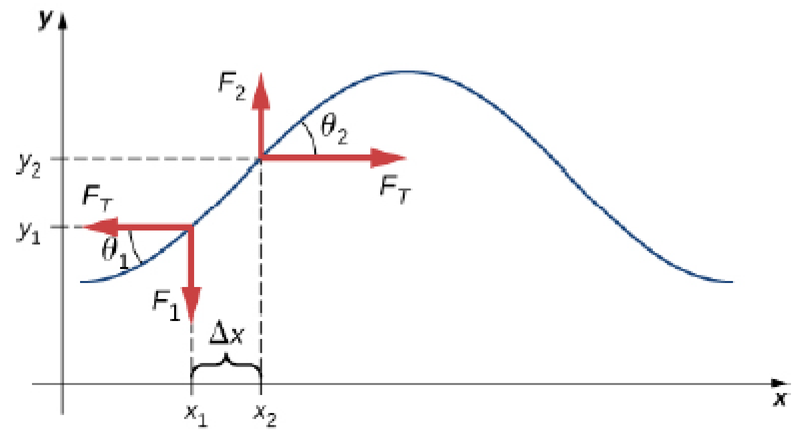


Figure 16.14 A string under tension is plucked, causing a pulse to move along the string in the positive *x*-direction.

The derivation of the wave equation for a string under tension (pp 824-825) is rather complicated, but we can qualitatively explain the result.

$$\frac{\partial^2 y(x,\, t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x,\, t)}{\partial t^2}.$$

Therefore,

$$\frac{1}{v^2} = \frac{\mu}{F_T}.$$

Solving for v, we see that the speed of the wave on a string depends on the tension and the linear density.

Speed of a Wave on a String Under Tension

The speed of a pulse or wave on a string under tension can be found with the equation

$$|v| = \sqrt{\frac{F_T}{\mu}} \tag{16.8}$$

where F_T is the tension in the string and μ is the mass per length of the string.

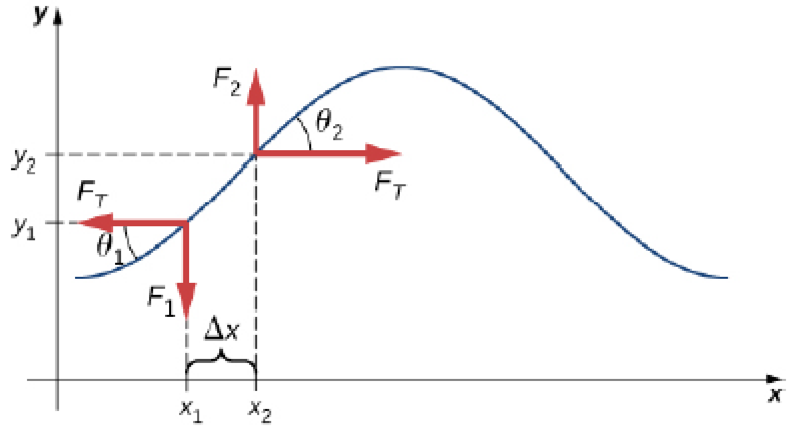


Figure 16.14 A string under tension is plucked, causing a pulse to move along the string in the positive *x*-direction.

The derivation of the wave equation for a string under tension (pp 824-825) is rather complicated, but we can qualitatively explain the result.

$$\boxed{\frac{\partial^2 y(x, t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x, t)}{\partial t^2}.}$$

Linear wave equation. One of the most important equations in physics/engineering!

Therefore,

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Speed of a Wave on a String Under Tension

The speed of a pulse or wave on a string under tension can be found with the equation

$$|v| = \sqrt{\frac{F_T}{\mu}} \qquad (16.8)$$

where F_T is the tension in the string and μ is the mass per length of the string.

In general, the speed of a wave in a medium is proportional to the square root of the ratio of elastic to inertial properties:

Speed of Compression Waves in a Fluid

The speed of a wave on a string depends on the square root of the tension divided by the mass per length, the linear density. In general, the speed of a wave through a medium depends on the elastic property of the medium and the inertial property of the medium.

$$|v| = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}}$$

The elastic property describes the tendency of the particles of the medium to return to their initial position when perturbed. The inertial property describes the tendency of the particle to resist changes in velocity.

The speed of a longitudinal wave through a liquid or gas depends on the density of the fluid and the bulk modulus of the fluid,

$$v = \sqrt{\frac{B}{\rho}}. \tag{16.9}$$

Here the bulk modulus is defined as $B = -\frac{\Delta P}{\frac{\Delta V}{V_0}}$, where ΔP is the change in the pressure and the denominator is the ratio of the change in volume to the initial volume, and $\rho \equiv \frac{m}{V}$ is the mass per unit volume. For example, sound is a mechanical wave that travels through a fluid or a solid. The speed of sound in air with an atmospheric pressure of 1.013×10^5 Pa and a temperature of $20°\text{C}$ is $v_s \approx 343.00$ m/s. Because the density depends on temperature, the speed of sound in air depends on the temperature of the air. This will be discussed in detail in Sound.