Physics 121 – November 2, 2017

Announcements:

Costa Concordia timelapse video

Interesting NRAO seminar tomorrow!

Assignments:

This week:

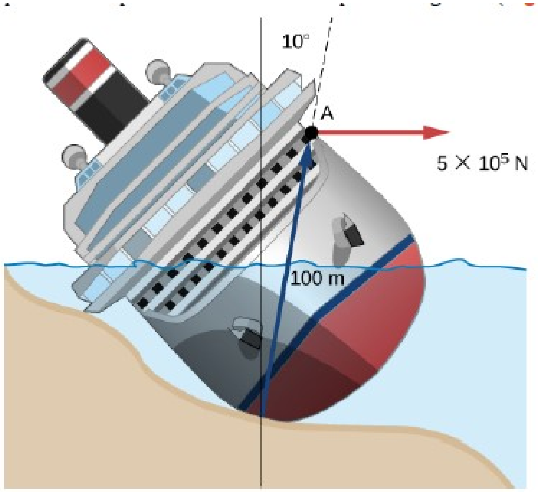
- Read Chapter 11.
- Complete ETA Problem Set #11 by Monday, Nov 6.

End-of-chapter problems: Ch 10: 71, 84, 104; Ch 11: 48, 53, 62. Due by 4 pm, Nov 6.

- Recitation: Practice problems on torque and angular momentum

Topics for today:

- Revisit the Costa Concordia example. Diagrams of the salvage are here.

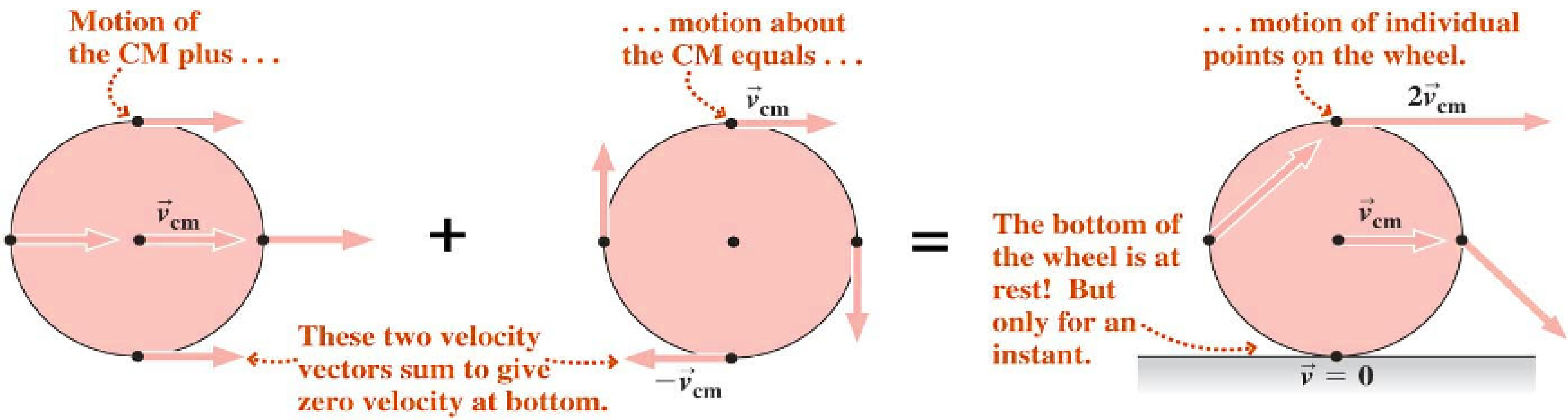


- Review rolling motion
- Look at the ETA push-up problem.
- Work and power in rotational motion
- Angular momentum

We should spend some time looking at ETA problem 10.8.4.

Note that our results imply the possibility of a “floating push up” (or the Planche exercise in gymnastics)

Rolling motion



Key points:

$v_{cm} = \omega R$ in ground reference frame

$v_{tang} = \omega R$ in wheel reference frame

$v = 0$ for contact point in ground reference frame

$K_{total} = 1/2Mv_{cm}^2 + 1/2I\omega^2$

Work-Energy Theorem for Rotation

The work-energy theorem for a rigid body rotating around a fixed axis is

$$W_{AB} = K_B - K_A \tag{10.29}$$

where

$$K = \frac{1}{2} I \omega^2$$

and the rotational work done by a net force rotating a body from point A to point B is

$$W_{AB} = \int_{\theta_A}^{\theta_B} \left(\sum_i \tau_i \right) d\theta. \tag{10.30}$$

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torque!

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$$W_{AB} = \int_{\theta_A}^{\theta_B} \left(\sum_i \tau_i \right) d\theta. \tag{10.30}$$

If the net torque is constant, then

$$P = \frac{dW}{dt} = \frac{d}{dt}(\tau\theta) = \tau \frac{d\theta}{dt}$$

$$P = \tau\omega.$$

Example 10.19

Torque on a Boat Propeller

Note: 9 x 10^4 W = 120 hp, a typical outboard engine size

A boat engine operating at 9.0×10^4 W is running at 300 rev/min. What is the torque on the propeller shaft?

Strategy

We are given the rotation rate in rev/min and the power consumption, so we can easily calculate the torque.

Solution

$$300.0 \text{ rev/min} = 31.4 \text{ rad/s};$$

$$\tau = \frac{P}{\omega} = \frac{9.0 \times 10^4 \text{ N} \cdot \text{m/s}}{31.4 \text{ rad/s}} = 2864.8 \text{ N} \cdot \text{m}.$$

Significance

It is important to note the radian is a dimensionless unit because its definition is the ratio of two lengths. It therefore does not appear in the solution.

10.8 Check Your Understanding A constant torque of $500 \text{ kN} \cdot \text{m}$ is applied to a wind turbine to keep it rotating at 6 rad/s. What is the power required to keep the turbine rotating?

Angular Momentum of a Particle

The **angular momentum** $\vec{\mathbf{l}}$ of a particle is defined as the cross-product of $\vec{\mathbf{r}}$ and $\vec{\mathbf{p}}$, and is perpendicular to the plane containing $\vec{\mathbf{r}}$ and $\vec{\mathbf{p}}$:

$$\vec{\mathbf{l}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}. \tag{11.5}$$

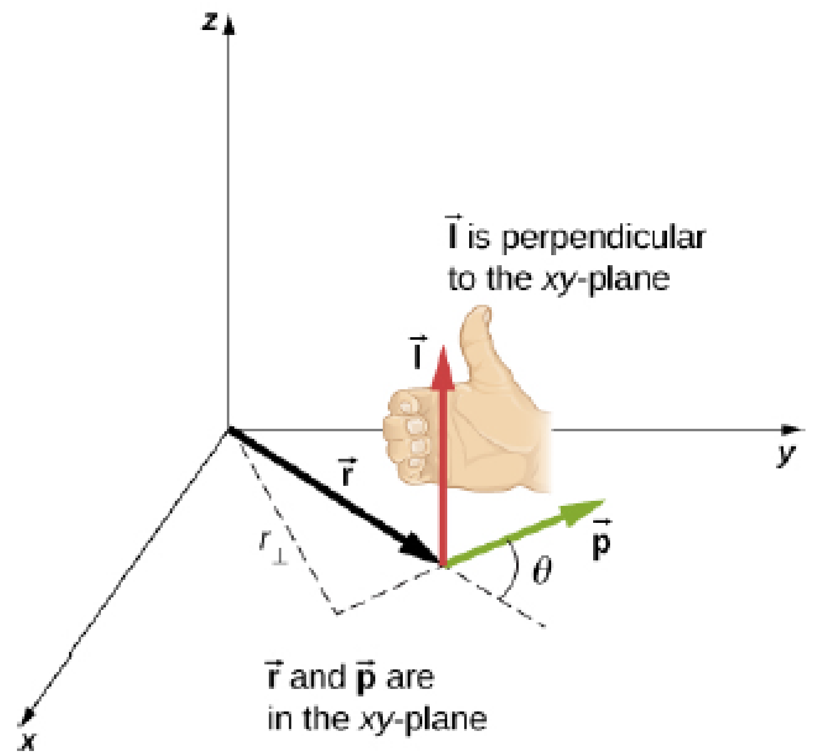


Figure 11.9 In three-dimensional space, the position vector $\vec{\mathbf{r}}$ locates a particle in the *xy*-plane with linear momentum $\vec{\mathbf{p}}$. The angular momentum with respect to the origin is $\vec{\mathbf{l}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}$, which is in the *z*-direction. The direction of $\vec{\mathbf{l}}$ is given by the right-hand rule, as shown.

For a rigid body, we add up all of the mass elements to find the net angular momentum

The net angular momentum of the rigid body along the axis of rotation is

$$L = \sum_i (\vec{\mathrm{T}}_i)_z = \sum_i R_i \Delta m_i v_i = \sum_i R_i \Delta m_i (R_i \omega) = \omega \sum_i \Delta m_i (R_i)^2 .$$

The summation $\sum_i \Delta m_i (R_i)^2$ is simply the moment of inertia I of the rigid body about the axis of rotation. For a thin hoop rotating about an axis perpendicular to the plane of the hoop, all of the R_i's are equal to R so the summation reduces to $R^2 \sum_i \Delta m_i = mR^2$, which is the moment of inertia for a thin hoop found in Figure 10.20. Thus, the magnitude of the angular momentum along the axis of rotation of a rigid body rotating with angular velocity ω about the axis is

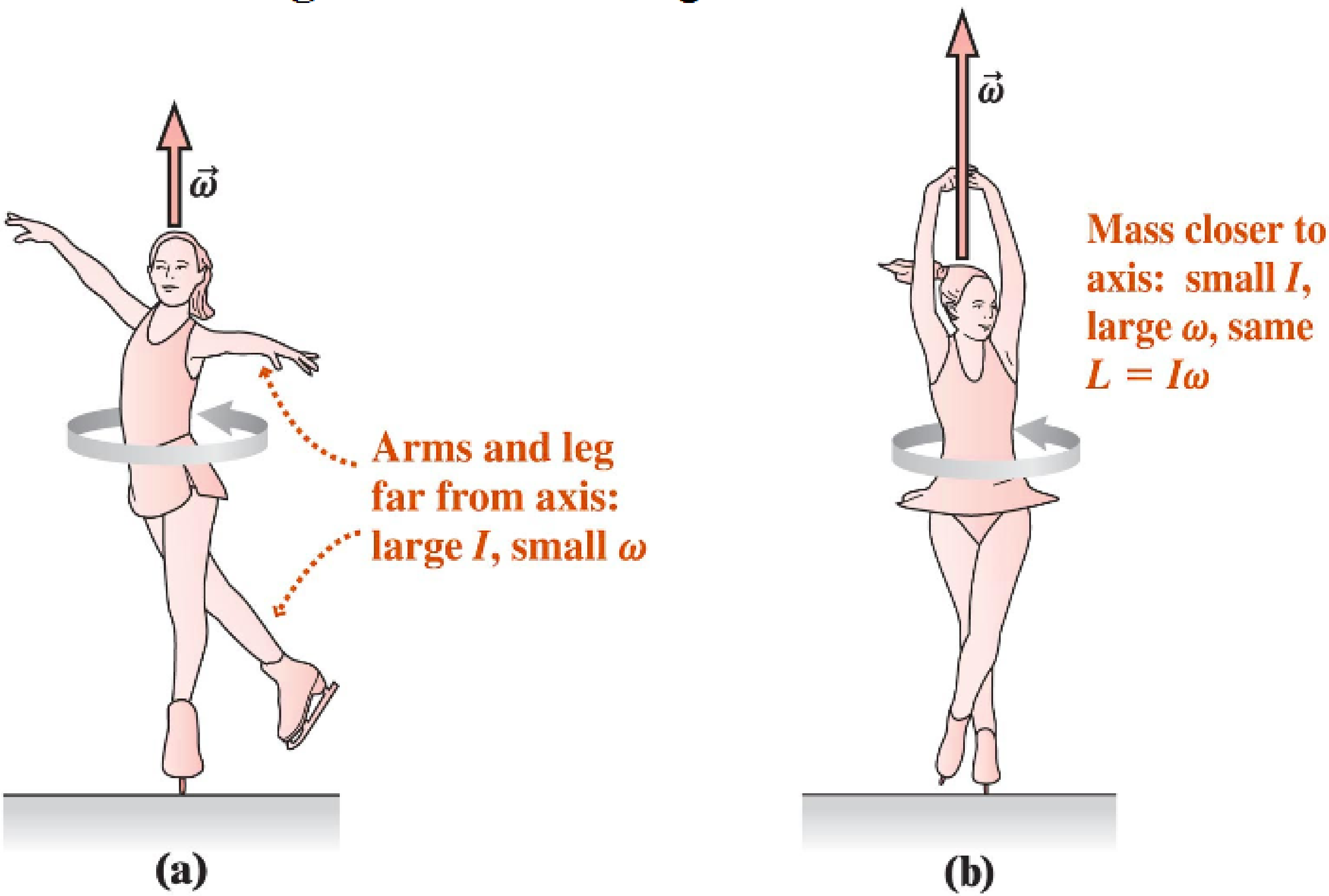
$$L = I\omega. \tag{11.9}$$

The direction of **L** is the same as the direction of $\boldsymbol{\omega}$

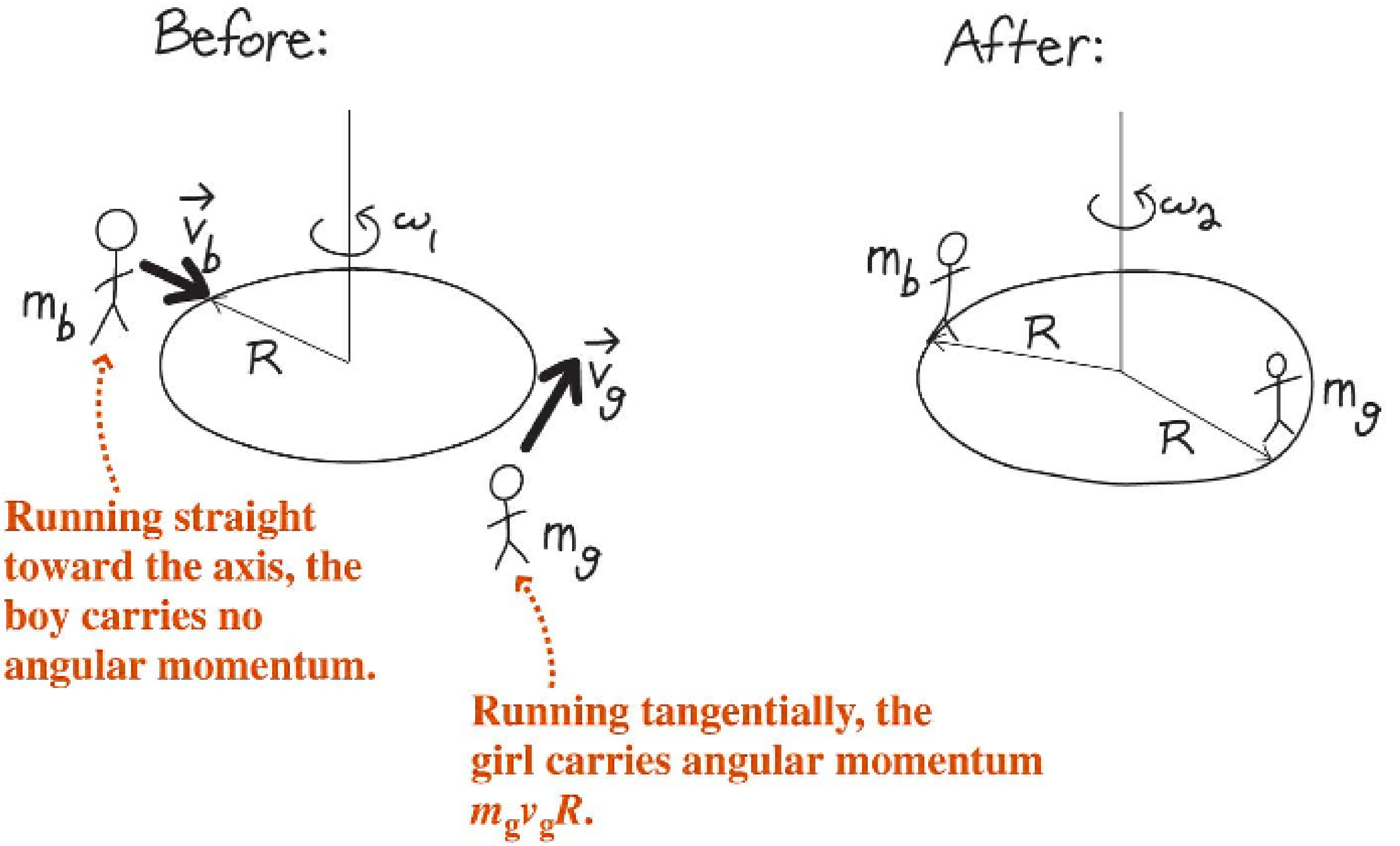
Some problems that use conservation of angular momentum involve a changing moment of inertia, I.

L = I**ω** = constant

The *magnitude* of **ω** changes, but not its direction.

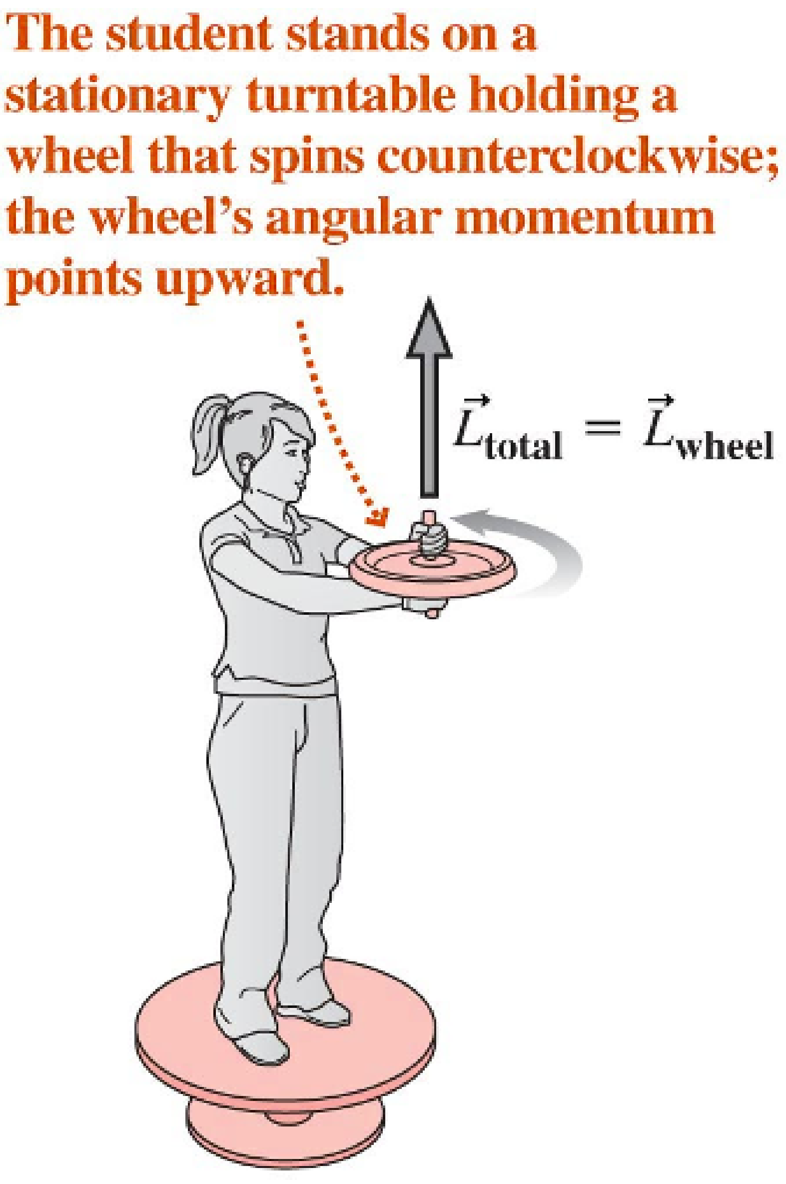


Other problems may involve multiple rotating elements with a single system. If the sum of the external torques on the system is zero, then **L** *for the entire system* must remain constant.



$$L(initial) = I\omega_1 + m_g v_g R$$
$$L(final) = [I + (m_g + m_b)R^2]\omega_2$$

This example is similar to the "rotational collision" lab exercise.

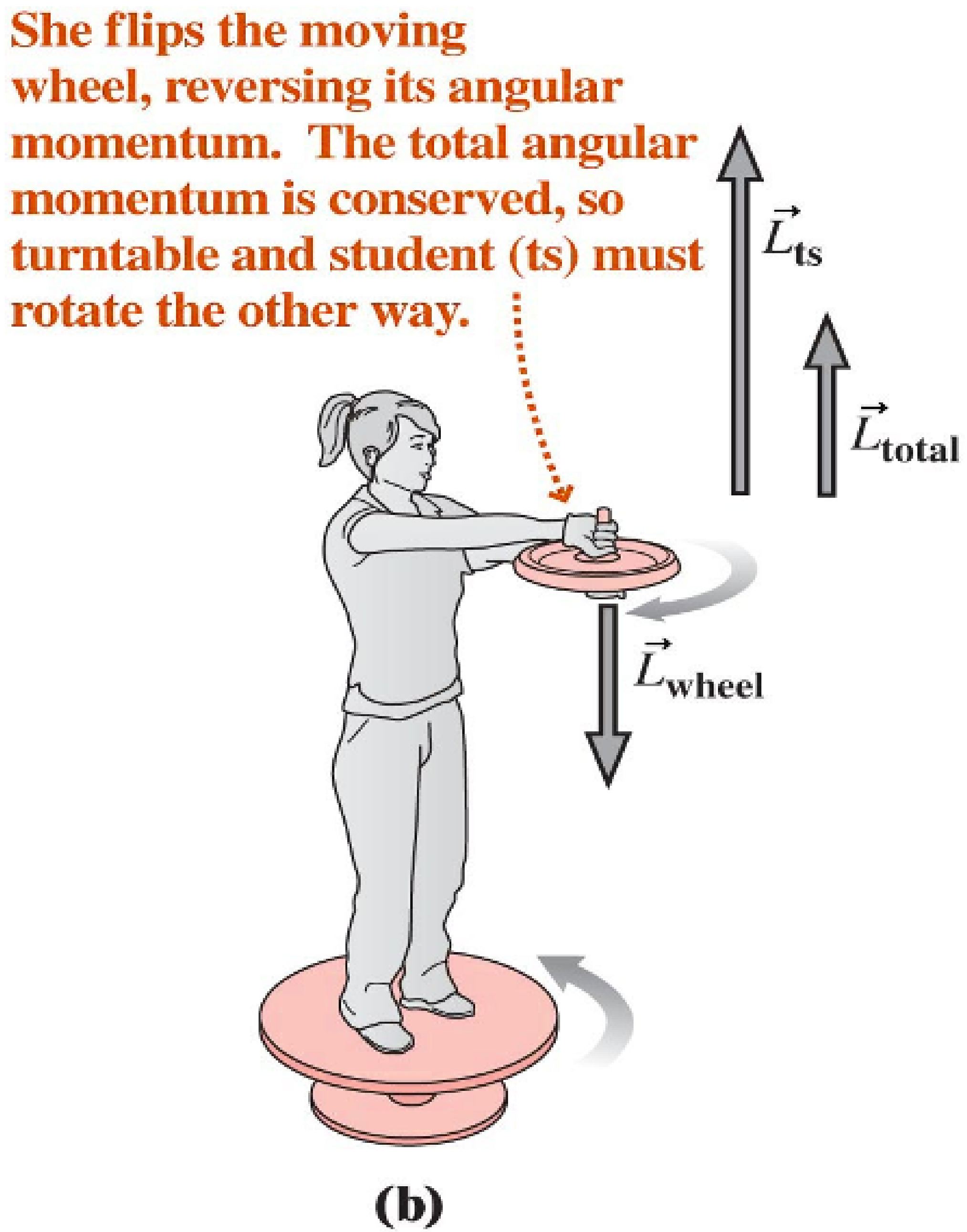


(a)

Iclicker question:

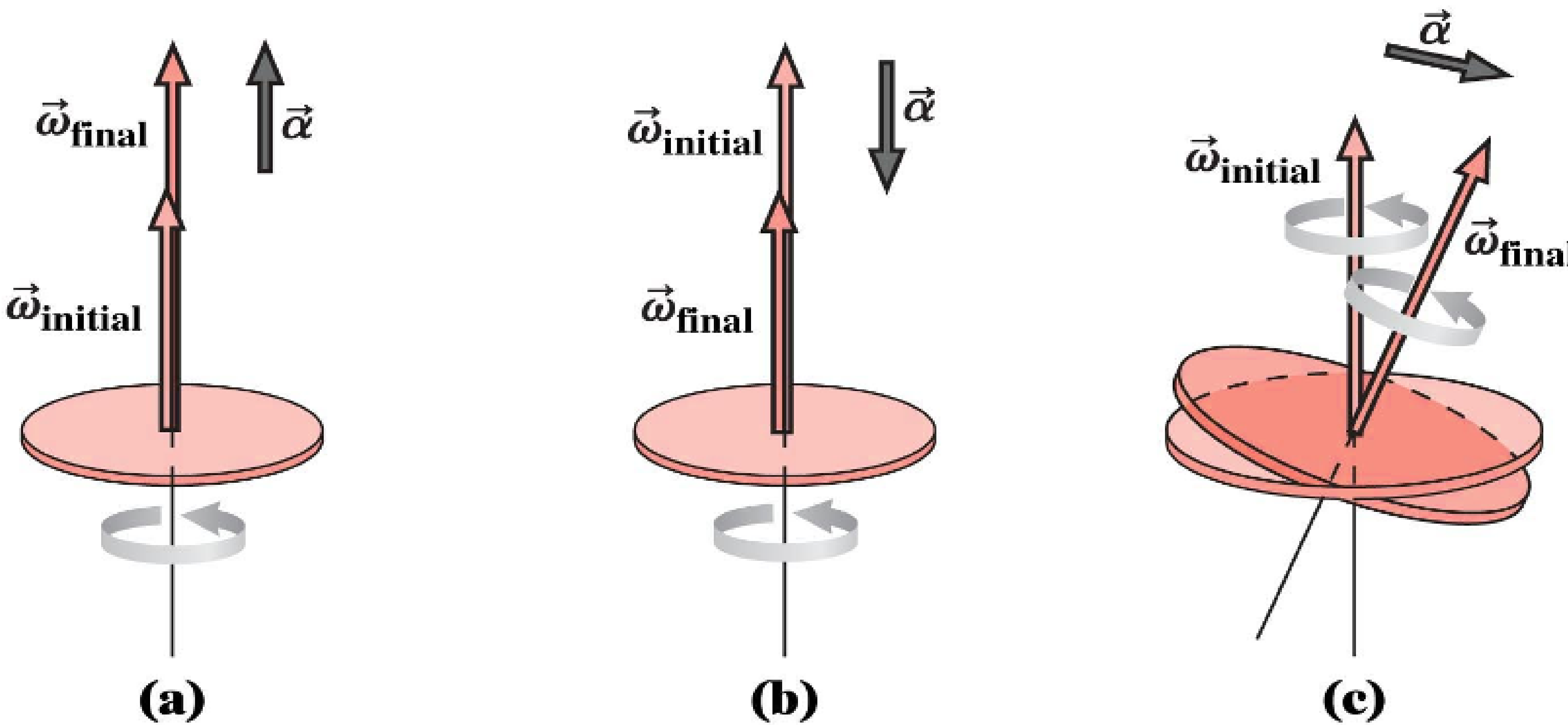
What happens to the turntable if the student flips the wheel upside down while it is spinning?

A. Turntable does nothing.
B. Turntable rotates clockwise.
C. Turntable rotates counter-clockwise.
D. Question is moot. Student is not capable of flipping wheel.



Note: two axes of rotation here (wheel, turntable)

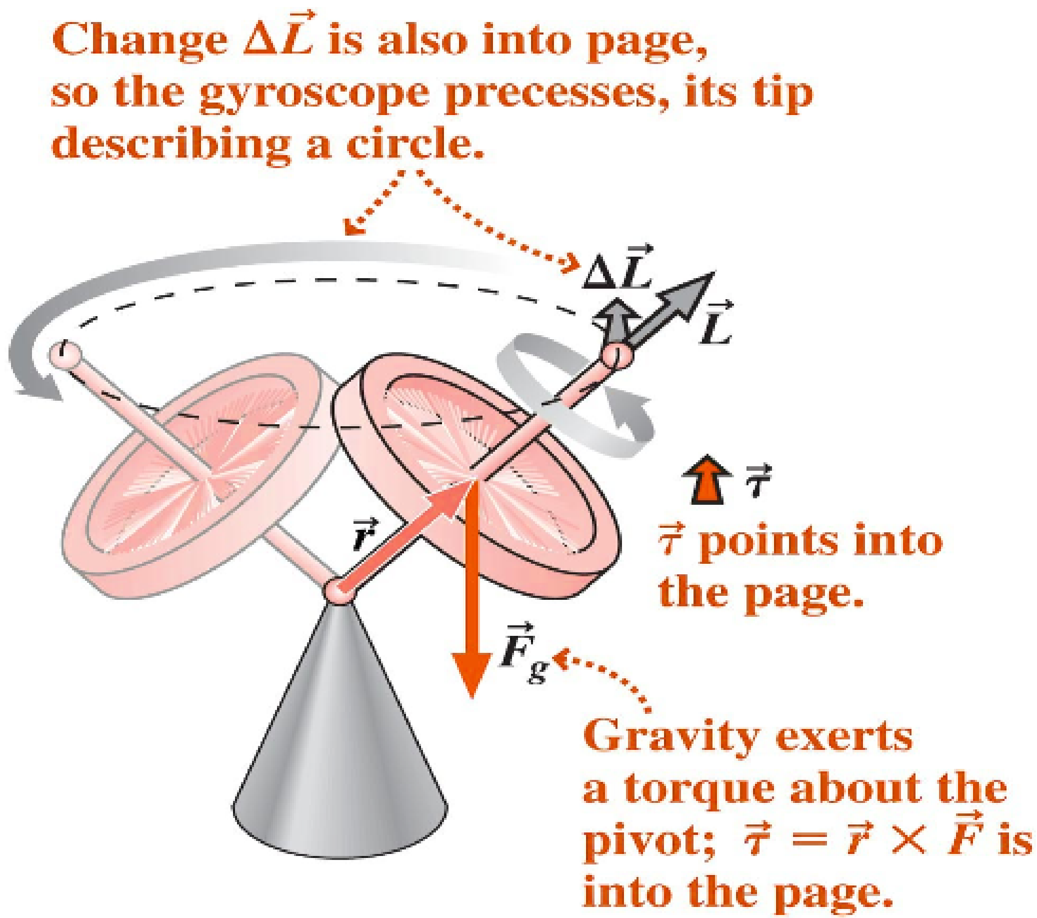
Sometimes a change in angular momentum results from a change in the *direction* of **ω**



$$\vec{\alpha} = \frac{d\,\vec{\omega}}{d\,t}$$

Magnitude and direction of **α** is given by the *change* in **ω**

Now, consider a system where there are two axes of rotation, but *they are not coincident*.



$$\frac{d\vec{L}}{dt} = \vec{\tau}$$

Change in *magnitude* or *direction* of total **L** of the gyroscope is given by **τ**