# CHAPTER 3 REVIEW

Vectors
Units & conversions
~~FBDs~~
Concepts, principles, law
Tools < diagrams / equations

## KEY TERMS

**acceleration due to gravity** acceleration of an object as a result of gravity

**average acceleration** the rate of change in velocity; the change in velocity over time

**average speed** the total distance traveled divided by elapsed time

**average velocity** the displacement divided by the time over which displacement occurs

**displacement** the change in position of an object

**distance traveled** the total length of the path traveled between two positions

**elapsed time** the difference between the ending time and the beginning time

**free fall** the state of movement that results from gravitational force only

**instantaneous acceleration** acceleration at a specific point in time

**instantaneous speed** the absolute value of the instantaneous velocity

**instantaneous velocity** the velocity at a specific instant or time point

**kinematics** the description of motion through properties such as position, time, velocity, and acceleration

**position** the location of an object at a particular time

**total displacement** the sum of individual displacements over a given time period

**two-body pursuit problem** a kinematics problem in which the unknowns are calculated by solving the kinematic equations simultaneously for two moving objects

## KEY EQUATIONS

These words are equally important.

| | |
|---|---|
| Displacement | $\Delta x = x_{\text{f}} - x_{\text{i}}$ |
| Total displacement | $\Delta x_{\text{Total}} = \sum \Delta x_{\text{i}}$ |
| Average velocity | $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$ |
| Instantaneous velocity | $v(t) = \frac{dx(t)}{dt}$ |
| Average speed | Average speed $= \bar{s} = \frac{\text{Total distance}}{\text{Elapsed time}}$ |
| Instantaneous speed | Instantaneous speed $= \lvert v(t) \rvert$ |
| Average acceleration | $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}$ |
| Instantaneous acceleration | $a(t) = \frac{dv(t)}{dt}$ |
| Position from average velocity | $x = x_0 + \bar{v}t$ |
| Average velocity | $\bar{v} = \frac{v_0 + v}{2}$ |
| Velocity from acceleration | $v = v_0 + at$ (constant $a$) |

def'ns

* Kinematic eq'ns

| | |
|---|---|
| Position from velocity and acceleration | $x = x_0 + v_0 t + \frac{1}{2}at^2$ (constant $a$) |
| Velocity from distance | $v^2 = v_0^2 + 2a(x - x_0)$ (constant $a$) |
| Velocity of free fall | $v = v_0 - gt$ (positive upward) |
| Height of free fall | $y = y_0 + v_0 t - \frac{1}{2}gt^2$ |
| Velocity of free fall from height | $v^2 = v_0^2 - 2g(y - y_0)$ |
| Velocity from acceleration | $v(t) = \int a(t)dt + C_1$ |
| Position from velocity | $x(t) = \int v(t)dt + C_2$ |

Kinematics for $a = -g$

by def'ns

## SUMMARY

### 3.1 Position, Displacement, and Average Velocity

- Kinematics is the description of motion without considering its causes. In this chapter, it is limited to motion along a straight line, called one-dimensional motion.
- Displacement is the change in position of an object. The SI unit for displacement is the meter. Displacement has direction as well as magnitude.
- Distance traveled is the total length of the path traveled between two positions.
- Time is measured in terms of change. The time between two position points $x_1$ and $x_2$ is $\Delta t = t_2 - t_1$. Elapsed time for an event is $\Delta t = t_f - t_0$, where $t_f$ is the final time and $t_0$ is the initial time. The initial time is often taken to be zero.
- Average velocity $\bar{v}$ is defined as displacement divided by elapsed time. If $x_1$, $t_1$ and $x_2$, $t_2$ are two position time points, the average velocity between these points is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}.$$

### 3.2 Instantaneous Velocity and Speed

- Instantaneous velocity is a continuous function of time and gives the velocity at any point in time during a particle's motion. We can calculate the instantaneous velocity at a specific time by taking the derivative of the position function, which gives us the functional form of instantaneous velocity $v(t)$.
- Instantaneous velocity is a vector and can be negative.
- Instantaneous speed is found by taking the absolute value of instantaneous velocity, and it is always positive.
- Average speed is total distance traveled divided by elapsed time.
- The slope of a position-versus-time graph at a specific time gives instantaneous velocity at that time.

### 3.3 Average and Instantaneous Acceleration

- Acceleration is the rate at which velocity changes. Acceleration is a vector; it has both a magnitude and direction. The SI unit for acceleration is meters per second squared.
- Acceleration can be caused by a change in the magnitude or the direction of the velocity, or both.
- Instantaneous acceleration $a(t)$ is a continuous function of time and gives the acceleration at any specific time during the motion. It is calculated from the derivative of the velocity function. Instantaneous acceleration is the slope of the velocity-versus-time graph.

# CHAPTER 4 REVIEW

## KEY TERMS

**acceleration vector** instantaneous acceleration found by taking the derivative of the velocity function with respect to time in unit vector notation

**angular frequency** $\omega$, rate of change of an angle with which an object that is moving on a circular path

**centripetal acceleration** component of acceleration of an object moving in a circle that is directed radially inward toward the center of the circle

**displacement vector** vector from the initial position to a final position on a trajectory of a particle

**position vector** vector from the origin of a chosen coordinate system to the position of a particle in two- or three-dimensional space

**projectile motion** motion of an object subject only to the acceleration of gravity

**range** maximum horizontal distance a projectile travels

**reference frame** coordinate system in which the position, velocity, and acceleration of an object at rest or moving is measured

**relative velocity** velocity of an object as observed from a particular reference frame, or the velocity of one reference frame with respect to another reference frame

**tangential acceleration** magnitude of which is the time rate of change of speed. Its direction is tangent to the circle.

**time of flight** elapsed time a projectile is in the air

**total acceleration** vector sum of centripetal and tangential accelerations

**trajectory** path of a projectile through the air

**velocity vector** vector that gives the instantaneous speed and direction of a particle; tangent to the trajectory

## KEY EQUATIONS

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|---|---|
| Position vector | $\vec{\mathbf{r}}\,(t) = x(t)\,\hat{\mathbf{i}} + y(t)\,\hat{\mathbf{j}} + z(t)\,\hat{\mathbf{k}}$ |
| Displacement vector | $\Delta\,\vec{\mathbf{r}} = \vec{\mathbf{r}}\,(t_2) - \vec{\mathbf{r}}\,(t_1)$ |
| Velocity vector | $\vec{\mathbf{v}}\,(t) = \lim\limits_{\Delta t \to 0} \dfrac{\vec{\mathbf{r}}\,(t+\Delta t) - \vec{\mathbf{r}}\,(t)}{\Delta t} = \dfrac{d\,\vec{\mathbf{r}}}{dt}$ |
| Velocity in terms of components | $\vec{\mathbf{v}}\,(t) = v_x(t)\,\hat{\mathbf{i}} + v_y(t)\,\hat{\mathbf{j}} + v_z(t)\,\hat{\mathbf{k}}$ * |
| Velocity components | $v_x(t) = \dfrac{dx(t)}{dt} \quad v_y(t) = \dfrac{dy(t)}{dt} \quad v_z(t) = \dfrac{dz(t)}{dt}$ |
| Average velocity | $\vec{\mathbf{v}}_{\text{avg}} = \dfrac{\vec{\mathbf{r}}\,(t_2) - \vec{\mathbf{r}}\,(t_1)}{t_2 - t_1}$ |
| Instantaneous acceleration | $\vec{\mathbf{a}}\,(t) = \lim\limits_{t \to 0} \dfrac{\vec{\mathbf{v}}\,(t+\Delta t) - \vec{\mathbf{v}}\,(t)}{\Delta t} = \dfrac{d\,\vec{\mathbf{v}}\,(t)}{dt}$ |
| Instantaneous acceleration, component form | $\vec{\mathbf{a}}\,(t) = \dfrac{dv_x(t)}{dt}\hat{\mathbf{i}} + \dfrac{dv_y(t)}{dt}\hat{\mathbf{j}} + \dfrac{dv_z(t)}{dt}\hat{\mathbf{k}}$ * |
| Instantaneous acceleration as second derivatives of position | $\vec{\mathbf{a}}\,(t) = \dfrac{d^2x(t)}{dt^2}\hat{\mathbf{i}} + \dfrac{d^2y(t)}{dt^2}\hat{\mathbf{j}} + \dfrac{d^2z(t)}{dt^2}\hat{\mathbf{k}}$ |

def'ns (not all independent)

| | |
|---|---|
| Time of flight | $T_{\text{tof}} = \dfrac{2(v_0 \sin\theta)}{g}$ |
| Trajectory | $y = (\tan\theta_0)x - \left[\dfrac{g}{2(v_0 \cos\theta_0)^2}\right]x^2$ |
| Range | $R = \dfrac{v_0^2 \sin 2\theta_0}{g}$ |

projectile motion

| | |
|---|---|
| Centripetal acceleration | $a_{\text{C}} = \dfrac{v^2}{r}$ |
| Position vector, uniform circular motion | $\vec{\mathbf{r}}(t) = A\cos\omega t\, \hat{\mathbf{i}} + A\sin\omega t\, \hat{\mathbf{j}}$ |
| Velocity vector, uniform circular motion | $\vec{\mathbf{v}}(t) = \dfrac{d\vec{\mathbf{r}}(t)}{dt} = -A\omega \sin\omega t\, \hat{\mathbf{i}} + A\omega\cos\omega t\, \hat{\mathbf{j}}$ |
| Acceleration vector, uniform circular motion | $\vec{\mathbf{a}}(t) = \dfrac{d\vec{\mathbf{v}}(t)}{dt} = -A\omega^2 \cos\omega t\, \hat{\mathbf{i}} - A\omega^2 \sin\omega t\, \hat{\mathbf{j}}$ |
| Tangential acceleration | $a_{\text{T}} = \dfrac{d\lvert\vec{\mathbf{v}}\rvert}{dt}$ |
| Total acceleration | $\vec{\mathbf{a}} = \vec{\mathbf{a}}_{\text{C}} + \vec{\mathbf{a}}_{\text{T}}$ |

circular motion

| | |
|---|---|
| Position vector in frame *S* is the position vector in frame $S'$ plus the vector from the origin of *S* to the origin of $S'$ | $\vec{\mathbf{r}}_{PS} = \vec{\mathbf{r}}_{PS'} + \vec{\mathbf{r}}_{S'S}$ |
| Relative velocity equation connecting two reference frames | $\vec{\mathbf{v}}_{PS} = \vec{\mathbf{v}}_{PS'} + \vec{\mathbf{v}}_{S'S}$ |
| Relative velocity equation connecting more than two reference frames | $\vec{\mathbf{v}}_{PC} = \vec{\mathbf{v}}_{PA} + \vec{\mathbf{v}}_{AB} + \vec{\mathbf{v}}_{BC}$ |
| Relative acceleration equation | $\vec{\mathbf{a}}_{PS} = \vec{\mathbf{a}}_{PS'} + \vec{\mathbf{a}}_{S'S}$ |

relative motion

# SUMMARY

## 4.1 Displacement and Velocity Vectors

- The position function $\vec{\mathbf{r}}(t)$ gives the position as a function of time of a particle moving in two or three dimensions. Graphically, it is a vector from the origin of a chosen coordinate system to the point where the particle is located at a specific time.
- The displacement vector $\Delta\vec{\mathbf{r}}$ gives the shortest distance between any two points on the trajectory of a particle in two or three dimensions.
- Instantaneous velocity gives the speed and direction of a particle at a specific time on its trajectory in two or three dimensions, and is a vector in two and three dimensions.
- The velocity vector is tangent to the trajectory of the particle.
- Displacement $\vec{\mathbf{r}}(t)$ can be written as a vector sum of the one-dimensional displacements $\vec{x}(t)$, $\vec{y}(t)$, $\vec{z}(t)$ along the *x*, *y*, and *z* directions.
- Velocity $\vec{\mathbf{v}}(t)$ can be written as a vector sum of the one-dimensional velocities $v_x(t)$, $v_y(t)$, $v_z(t)$ along the *x*, *y*, and *z* directions.

# CHAPTER 5 REVIEW

## KEY TERMS

**dynamics** study of how forces affect the motion of objects and systems

**external force** force acting on an object or system that originates outside of the object or system

**force** push or pull on an object with a specific magnitude and direction; can be represented by vectors or expressed as a multiple of a standard force

**free fall** situation in which the only force acting on an object is gravity

**free-body diagram** sketch showing all external forces acting on an object or system; the system is represented by a single isolated point, and the forces are represented by vectors extending outward from that point

**Hooke's law** in a spring, a restoring force proportional to and in the opposite direction of the imposed displacement

**inertia** ability of an object to resist changes in its motion

**inertial reference frame** reference frame moving at constant velocity relative to an inertial frame is also inertial; a reference frame accelerating relative to an inertial frame is not inertial

**law of inertia** see Newton's first law of motion

**net external force** vector sum of all external forces acting on an object or system; causes a mass to accelerate

**newton** SI unit of force; 1 N is the force needed to accelerate an object with a mass of 1 kg at a rate of $1 \text{ m/s}^2$

**Newton's first law of motion** body at rest remains at rest or, if in motion, remains in motion at constant velocity unless acted on by a net external force; also known as the law of inertia

**Newton's second law of motion** acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system and is inversely proportional to its mass

**Newton's third law of motion** whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that it exerts

**normal force** force supporting the weight of an object, or a load, that is perpendicular to the surface of contact between the load and its support; the surface applies this force to an object to support the weight of the object

**tension** pulling force that acts along a stretched flexible connector, such as a rope or cable

**thrust** reaction force that pushes a body forward in response to a backward force

**weight** force $\vec{\mathbf{w}}$ due to gravity acting on an object of mass $m$

## KEY EQUATIONS

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|---|---|
| Net external force | $\vec{\mathbf{F}}_{\text{net}} = \sum \vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \cdots$ |
| Newton's first law | $\vec{\mathbf{v}} = \text{constant when } \vec{\mathbf{F}}_{\text{net}} = \vec{\mathbf{0}} \text{ N}$ |
| Newton's second law, vector form | $\vec{\mathbf{F}}_{\text{net}} = \sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$ |
| Newton's second law, scalar form | $F_{\text{net}} = ma$ |
| Newton's second law, component form | $\sum \vec{\mathbf{F}}_x = m\vec{\mathbf{a}}_x, \sum \vec{\mathbf{F}}_y = m\vec{\mathbf{a}}_y, \text{ and } \sum \vec{\mathbf{F}}_z = m\vec{\mathbf{a}}_z.$ |
| Newton's second law, momentum form | $\vec{\mathbf{F}}_{\text{net}} = \frac{d\vec{\mathbf{p}}}{dt}$ , $\vec{p} = m\vec{v}$ |
| Definition of weight, vector form | $\vec{\mathbf{w}} = m\vec{\mathbf{g}}$ |