PHYSICS 570 – Master's of Science Teaching

"Electricity"
Lecture 12 – Resistance and Resistivity

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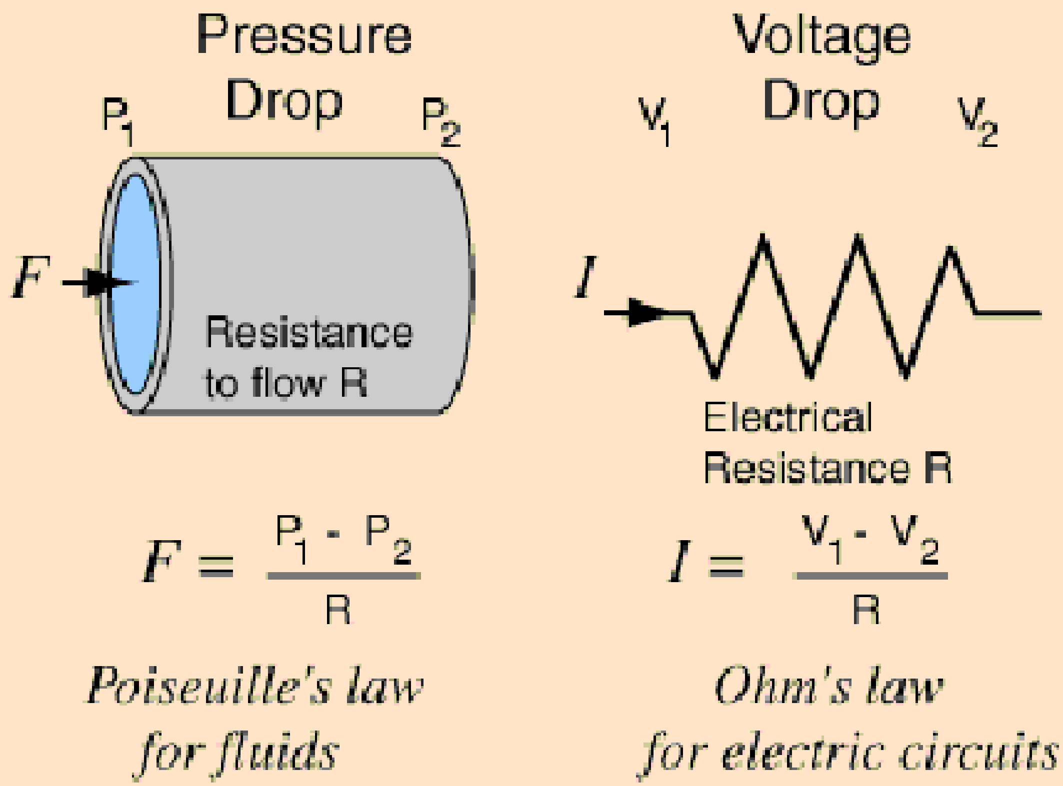
Ohm's Law-Poiseuille's Law

Ohm's law for electric current flow and Poiseuille's law for the smooth flow of fluids are of the same form.



Water analogy to DC circuits

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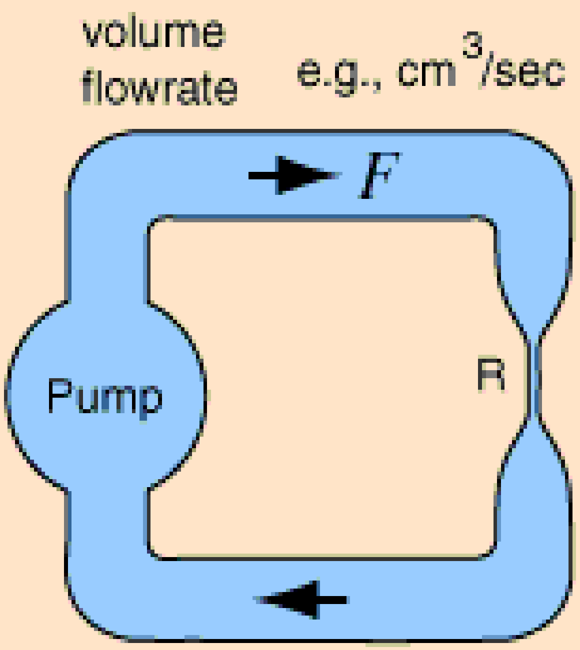
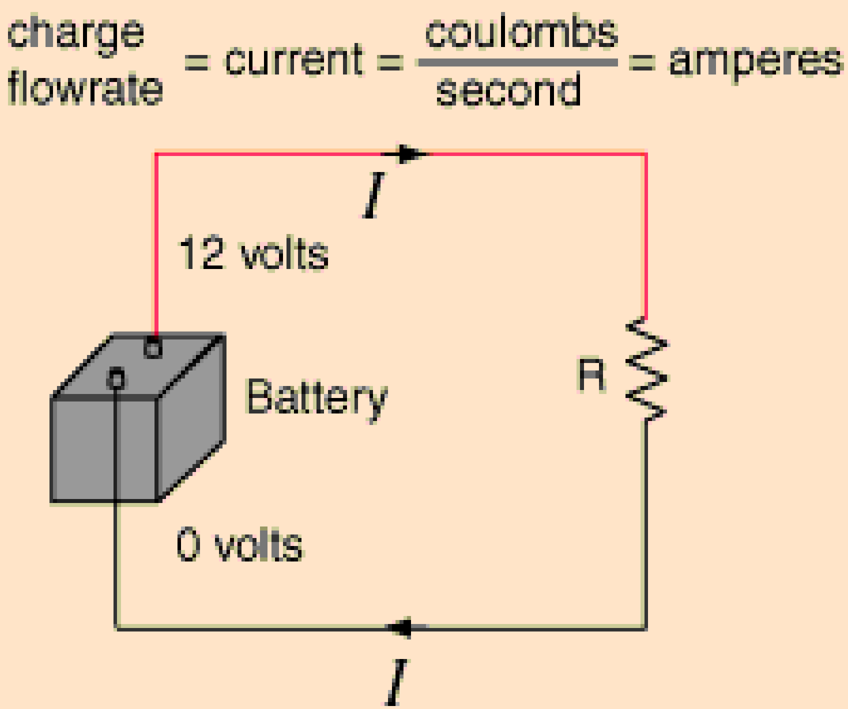
DC Circuits

Current Law and Flowrate



With continuous circulation around the pipe system, the volume flowrate must be the same at any cross-section of the pipe system.

Conservation of liquid



The electric current is the charge flowrate and it must be the same at any cross-section of the circuit. This is a general principle called the current law.

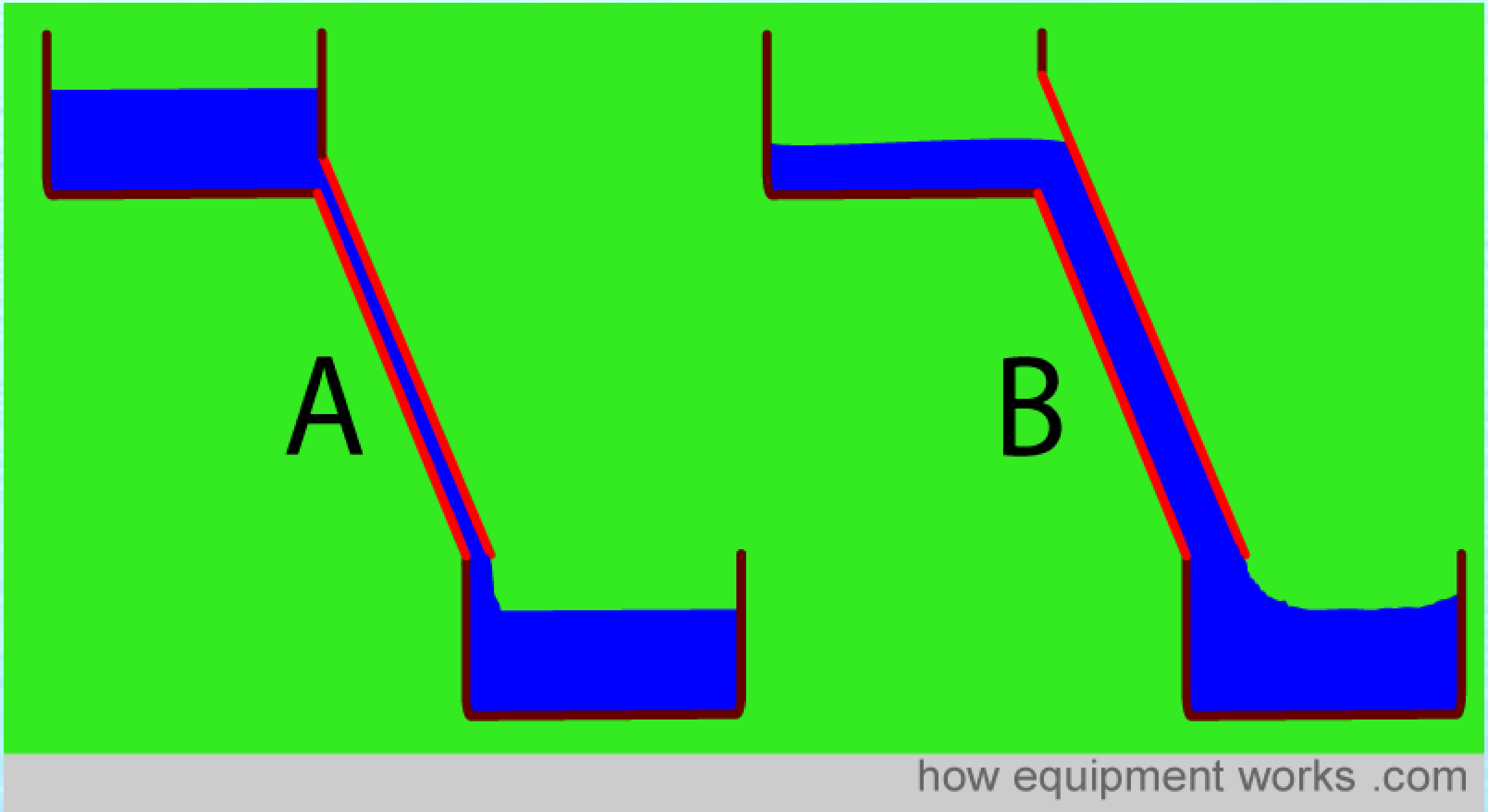
Conservation of charge

Hydraulic Analogy

http://en.wikipedia.org/wiki/Hydraulic_analogy

Mass of water (M)	**Charge (Q)**
Flow Rate (F) (dM/dt)	**Current (I = dQ/dt)**
Water pressure (p)	**Voltage (V)**
Volume of Tank (ν)	**Capacitance (C)**
Resistance of hose (R)	**Resistance (R)**

How should resistance depend on length and area of a wire?



How should resistance depend on length and area of a wire?

A narrow hose passes less water at a certain pressure difference than a large hose.

$$R \simeq \frac{1}{A}$$

A long hose passes less water at a certain pressure than a short hose.

$$R \simeq L$$

How should resistance depend on length and area of a wire?

A “frictiony” (rough on the inside) hose should carry less water than a smooth hose. Let's call “rho” the friction coefficient.

$$R \simeq \rho$$

Now put it all together.

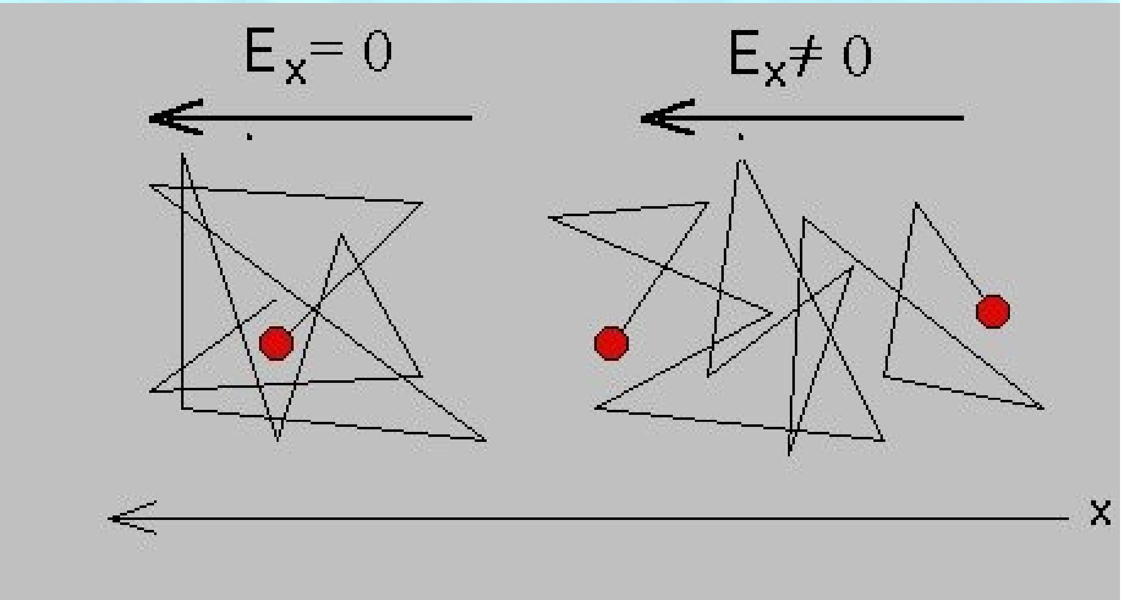
$$R = \rho \frac{L}{A}$$

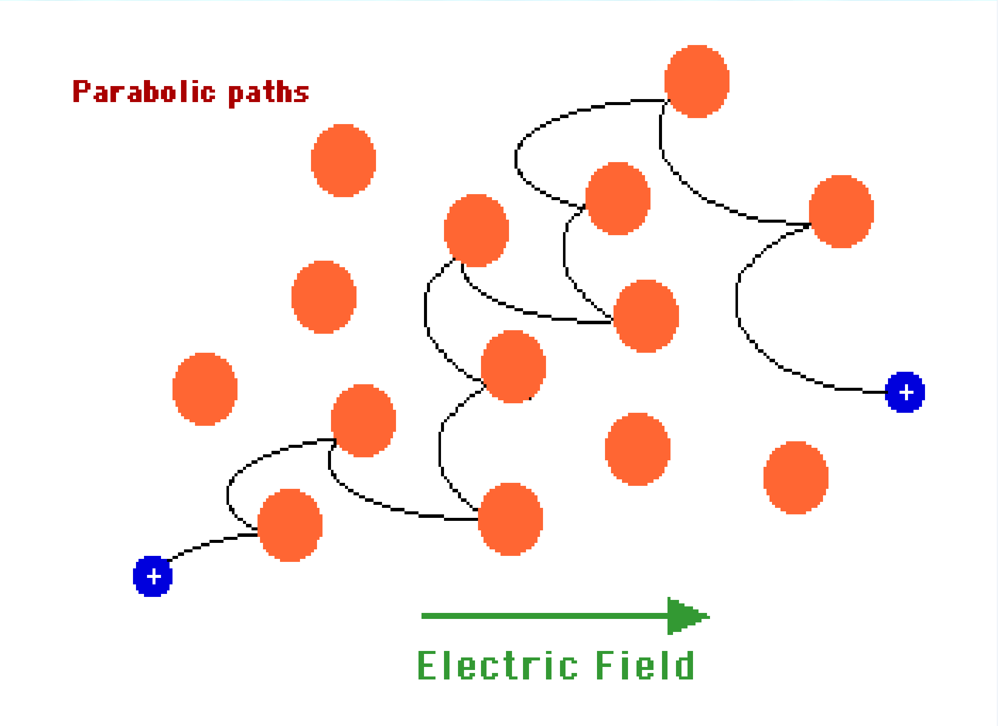
BONUS – SKIP IF YOU WANT:

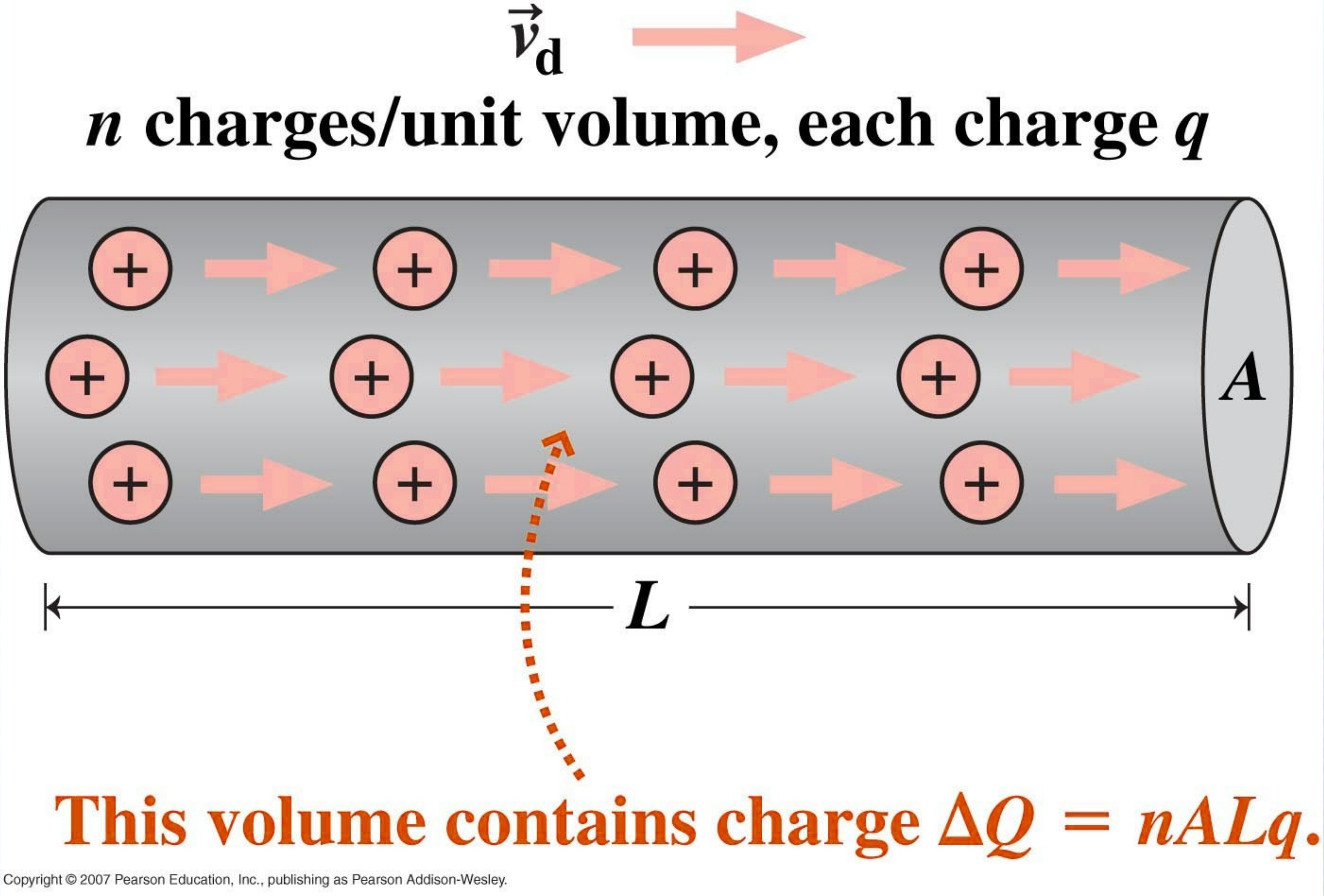
Microscopic view of resistance

"Free electron gas" model (also called Drude model) of a metal.

You can derive Ohm's law by assuming a metal is a box full of loose electrons that bump into "scattering centers" every τ trillionth of a second.

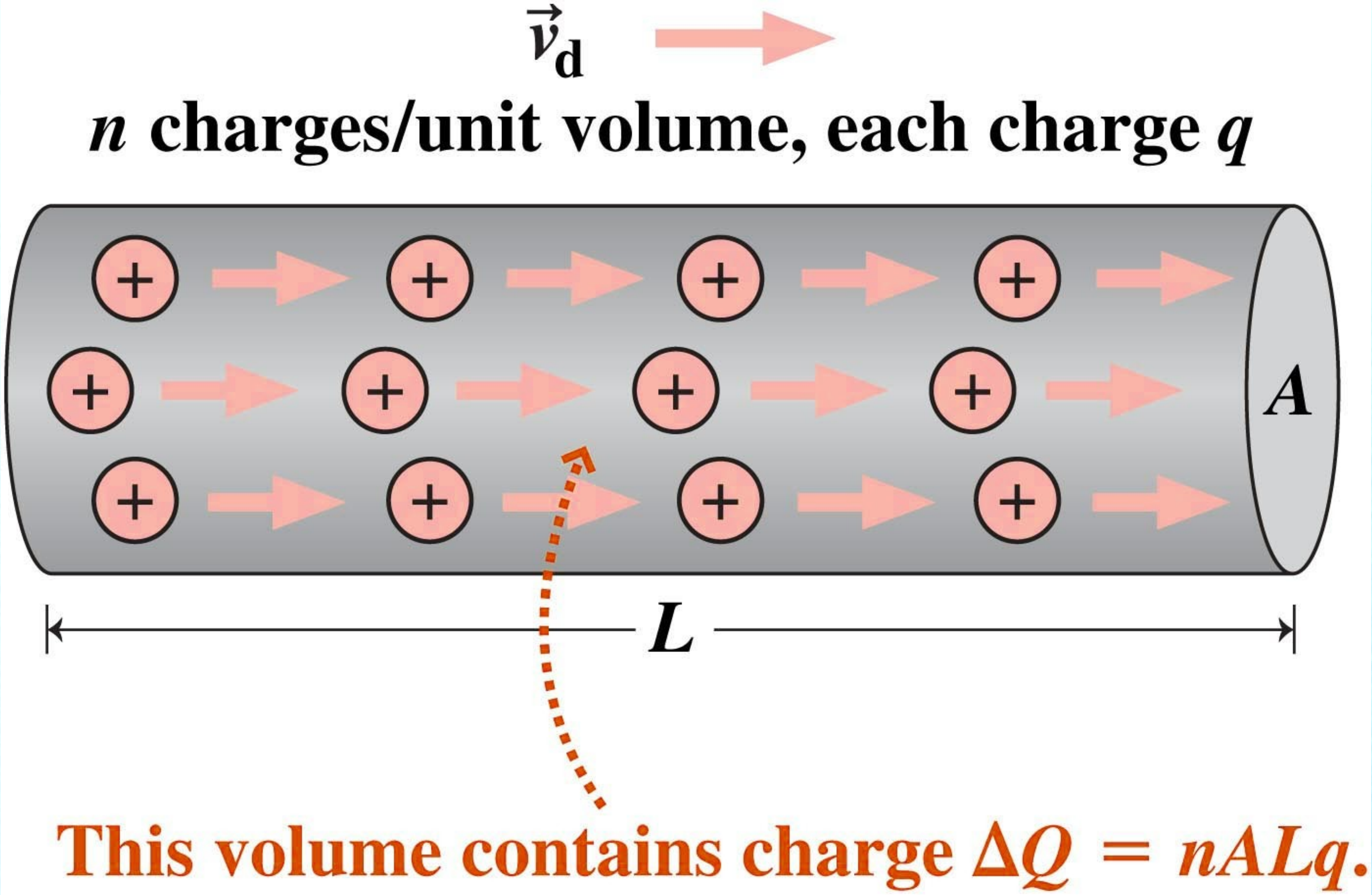






$$\Delta Q = nA\vec{v}_d \Delta t\ q$$

$$I = nAqv_d = nAqa\tau = \frac{nAq^2\tau}{m}E$$


$\vec{v}_d$
n charges/unit volume, each charge *q*
A
L
This volume contains charge $\Delta Q = nALq$.

BONUS – Deriving Ohm's law

$$I = n\,A\,q\,v_d = n\,A\,q\,a\,\tau = \frac{n\,A\,q^2\,\tau}{m}E$$

$$I = \frac{n\,A\,q^2\tau}{m}\frac{V}{L}$$

$$V = I\frac{m}{n\,q^2\,\tau}\frac{L}{A}$$

$$R = \frac{m}{n\,q^2\,\tau}\frac{L}{A}$$

$$\rho = \frac{m}{n\,q^2\,\tau}$$

BONUS – Deriving Ohm's law

$$\rho = \frac{m}{n\,q^2\,\tau}$$

Come on back... let's see what we learned

rho – resistivity … a property of each material.

m – Mass of an electron

tau – Time between collisions

q – Charge of an electron

n – number of electrons per volume

$$R = \frac{m}{n q^2 \tau} \frac{L}{A}$$

$$\rho = \frac{m}{n q^2 \tau}$$

Resistivity of common materials

$$R = \rho \frac{L}{A}$$

$$V = IR \quad \text{Ohm's Law}$$

$$\text{Copper}: \ \rho = 1.7 \times 10^{-8}\,\Omega \cdot m$$

$$\text{Tungsten}: \ \rho = 5.7 \times 10^{-8}\,\Omega \cdot m$$

$$\text{Nichrome}: \ \rho = 1.1 \times 10^{-6}\,\Omega \cdot m$$

$$\text{Sea Water}: \ \rho = 2.0 \times 10^{-1}\,\Omega \cdot m$$

$$\text{Silicon}: \ \rho = 6.4 \times 10^{2}\,\Omega \cdot m$$

$$\text{Glass}: \ \rho = 1.0 \times 10^{13}\,\Omega \cdot m$$

Applied Resistivity

$$R = \rho \frac{L}{A} \qquad \text{Copper: } \rho = 1.7 \times 10^{-8}\,\Omega \cdot \text{m}$$

What is resistance of a 50 foot extension cord made of 14-gauge wire.
14-gauge wire is about 2 mm in diameter.

(Homework 12-1)

Applied Resistivity

$$R = \rho \frac{L}{A} \quad \text{Nichrome: } \rho = 1.1 \times 10^{-6}\,\Omega \cdot m$$

You have a 1000 Watt toaster with nichrome wire. The wire is 0.5 mm in diameter. How long is it?

(Homework 12-2)

Jumper cables

$$R = \rho \frac{L}{A} \qquad \text{Copper: } \rho = 1.7 \times 10^{-8}\,\Omega \cdot m$$

You need to start a car with 15 foot jumper cables and you can only afford a 2 -V voltage drop through the cables. What should be the diameter of the copper? (The engine needs 500 Amps to crank over)

(Homework 12-3)