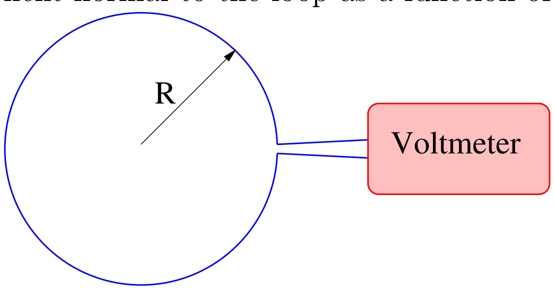
Physics 222 – Test 2 – Spring 2010

One-page reminder sheet allowed. *Show all work – no credit given if work not shown!* Note that $\mu_0 = 4\pi \times 10^{-7}$ N s^2 C^{-2}, $\epsilon_0 = 8.85 \times 10^{-12}$ C^2 N^{-1} m^{-2}, and that $c^2 = 1/(\epsilon_0\mu_0)$.

1. A circular loop of wire of radius R has a voltmeter connected to it as shown below. The voltmeter registers a voltage difference between the two ends of the loop equal to $\Delta V = V_0 \sin(\omega t)$ where V_0 and ω are constants.

   (a) Determine the magnetic flux Φ_B through the loop as a function of time.

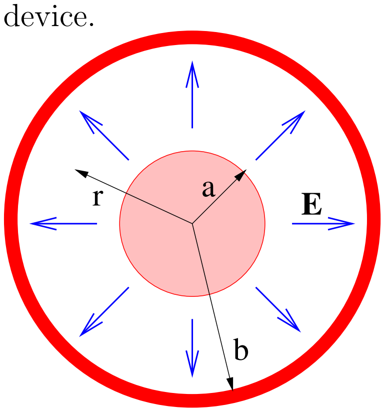
   (b) Assuming the magnetic field is uniform over the loop, determine the field component normal to the loop as a function of time.



2. A circular conducting pipe of length L has an internal radius b. A conducting rod of the same length and concentric with the pipe has radius a. Assume that $L \gg b$ so that end effects can be neglected.

   (a) If the electric field in the gap between the rod and the pipe points outward as shown, and depends on radius r according to $E = D/r$ where D is a positive constant, determine using Gauss's law the total charge on the inner rod.

   (b) Given the above $\mathbf{E}$ field, find the scalar potential in the gap between the rod and the inner surface of the pipe as a function of r. Then compute the potential difference between the rod and the cylinder and infer the capacitance of this device.



3. Determine the units of the electric and magnetic fields and the scalar and vector potential in terms of the fundamental SI units; meters (m), seconds (s), kilograms (kg), and coulombs (C). Hint: Start with the Lorentz force law and the definitions of the electric and magnetic fields in terms of the scalar and vector potentials.

4. In the static (time independent) case show that the scalar potential inside a conductor is the same everywhere in the conductor. Hint: What is the electric field in the conductor?

5. Suppose in a given situation the four-potential is $\underline{a} = (A_x, 0, 0, \phi/c)$ where A_x and ϕ are positive and where $\phi > cA_x$.

   (a) Determine the speed and direction of the moving reference frame in which the vector potential is zero.

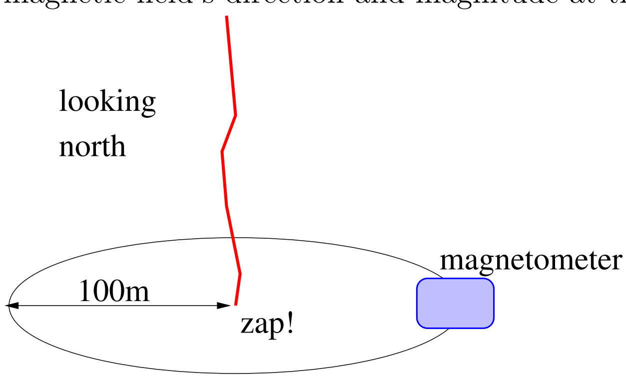
   (b) Determine the scalar potential in this reference frame.

6. A lightning flash lowers -10 C of charge vertically to the ground in 10^{-3} s.

   (a) What is the sign and magnitude of the current in the lightning flash over this period of time? Take a positive current as being upward.

   (b) Compute the magnetic circulation around the above current.

   (c) The magnetic field due to the lightning current is measured by a magnetometer 100 m to the east of where the lightning flash hits the ground. What is the magnetic field's direction and magnitude at the magnetometer?



7. The quantity

$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B}$$

   is called the Poynting vector.

   (a) For a free electromagnetic plane wave, show that $|\mathbf{S}| = cu$ where u is the electric plus magnetic energy density of the wave and c is the speed of light. Hint: Recall that $E = cB$ for a free electromagnetic wave.

   (b) The quantity cu is the magnitude of the energy flux carried by the wave. The energy flux in sunlight reaching the earth is about 1360 W m^{-2}. Determine the magnitudes of the electric and magnetic fields in sunlight at the earth's orbit.