Diffraction

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Radiation and Optics

Overview

- Basis for Diffraction
- Fraunhofer Diffraction
- Diffraction Gratings
- Fresnel Diffraction
- Fresnel Lenses

Diffraction

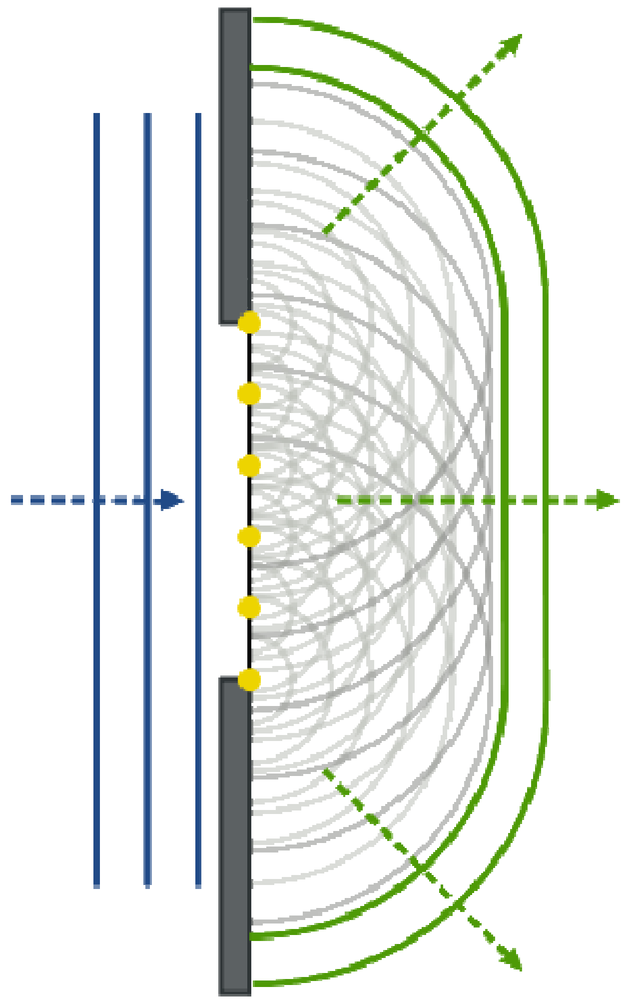
- The wave/physical optics behavior of any wave that encounters an obstacle
- It is closely related to interference in terms of how the downstream waves, after the obstacle, interact with one another
- Diffraction occurs when obstacle is of the order of the wavelength of the wave
- Sometimes we differentiate based on the number of sources: i.e. two slits = interference; many slits = diffraction

Diffraction History

- First carefully discussed by Grimaldi ~ 1665
- Term diffraction comes from Latin *diffringere* meaning to break into small pieces
- Others who studied it included Newton, Gregory, Young, Fresnel
- Large fight at the time as Newton was a proponent of corpuscular theory and Young re-invigorated wave theory with double-slit experiment
- Not until Einstein proposed wave-particle duality that this was settled

Huygens-Fresnel Principle

- Multiple points on the wavefront act as sources for the secondary wavelets
- These sources at the opening are able to create a wave which expands past the edge of the opening



Types of Diffraction

- Far-field – Called Fraunhofer
  - When diffraction pattern is viewed far from diffracting obstacle

$$\frac{W^2}{L\lambda} \ll 1$$

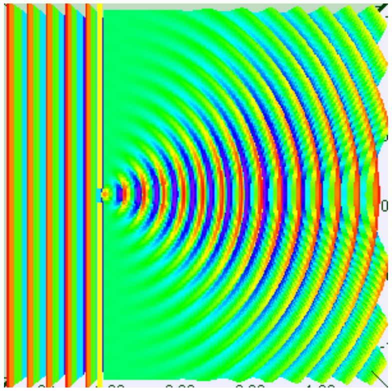
W –aperture size, L – distance away

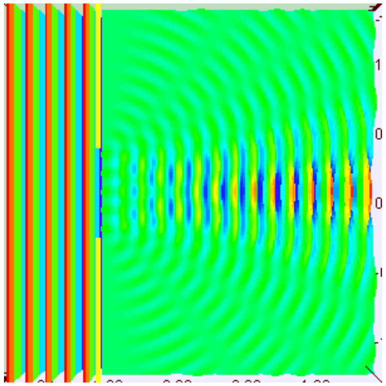
- Near-field – Called Fresnel
  - When diffraction pattern is viewed very close to diffracting obstacle

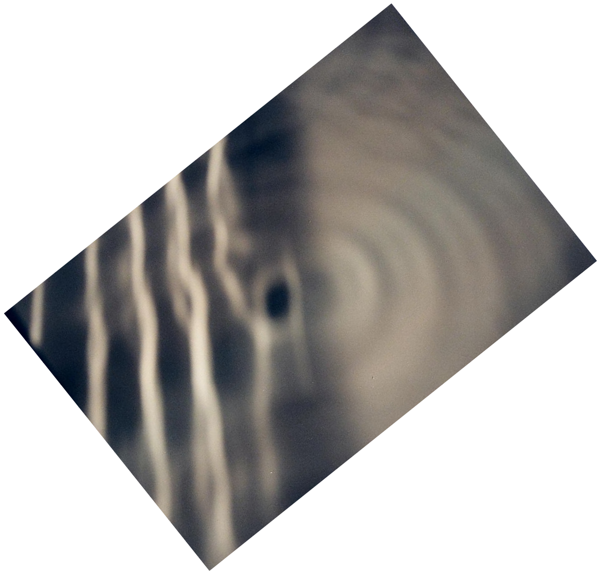
$$\frac{W^2}{L\lambda} \geq 1$$

Simplest – Single Slit Diffraction

- There is a downstream spreading of the wave for slits ~ wavelength of the wave
- As the slit gets wider, the diffraction pattern narrows
- Treat the slit as made up of multiple half-wavelength pieces

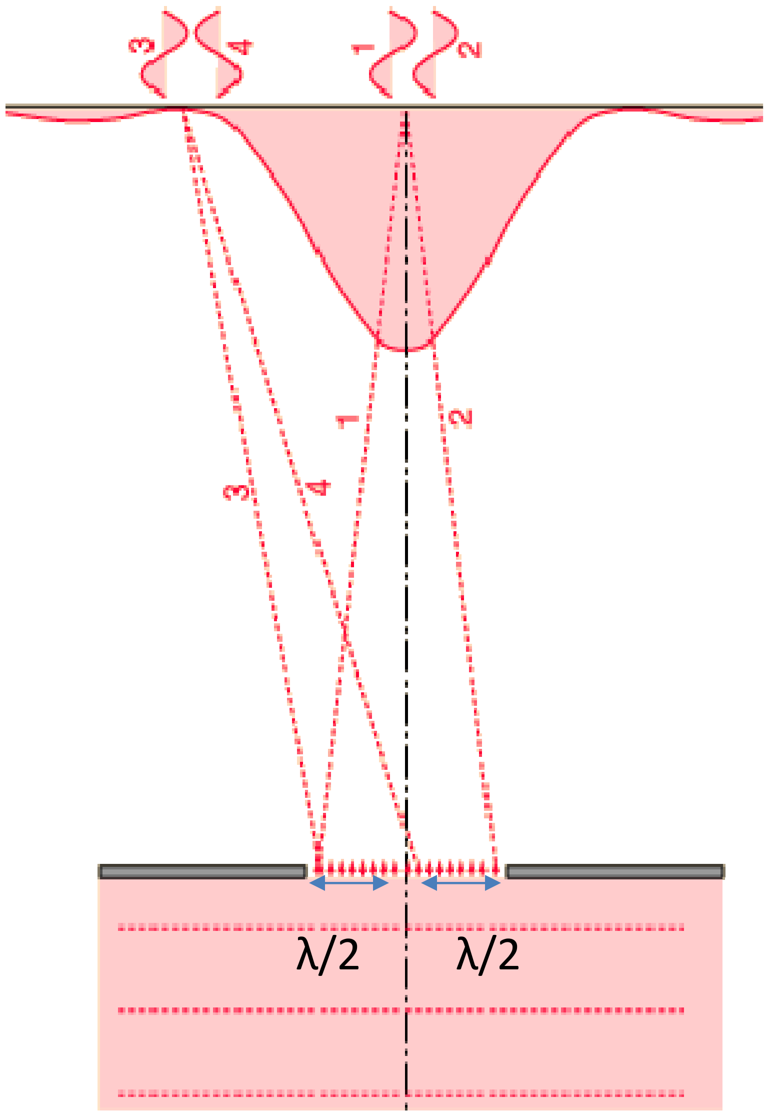






Fraunhofer Diffraction Concept

- Assume intensity is measured in the far field
- Break up the slit into λ/2 sections and look at how parts combine in the far field



The Calculations

- The path difference is given by: $\dfrac{d\sin\theta}{2}$
- Minimum intensity at an angle Θ_{min} given by:

$$d\sin\theta_{\min} = \lambda$$

- Divide the slit into an even number of n sections:

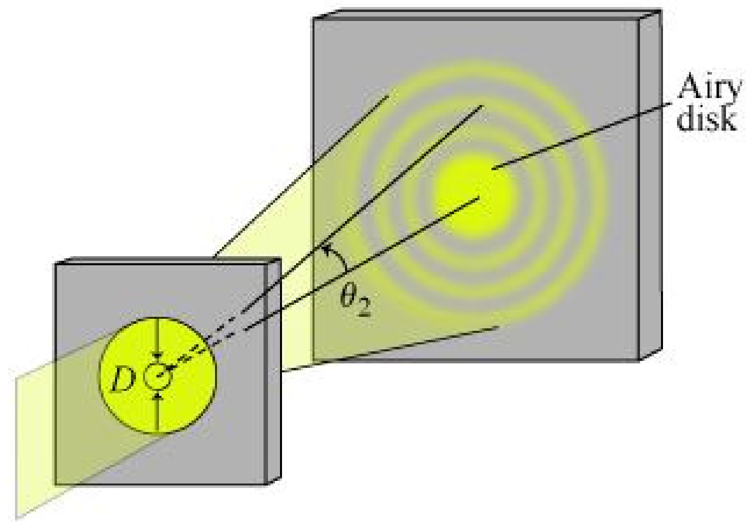
$$d\sin\theta_n = n\lambda$$

- From the Fraunhofer diffraction equation you can calculate the intensity:

$$I(\theta) = I_o \sin c^2\left(\frac{d\pi}{\lambda}\sin\theta\right)$$

A Circular Aperture

- Think of projecting your slit into a circle by spinning it about the midpoint
- You produce a diffraction pattern with a series of decreasing intensity rings around it
- This is called an Airy disk



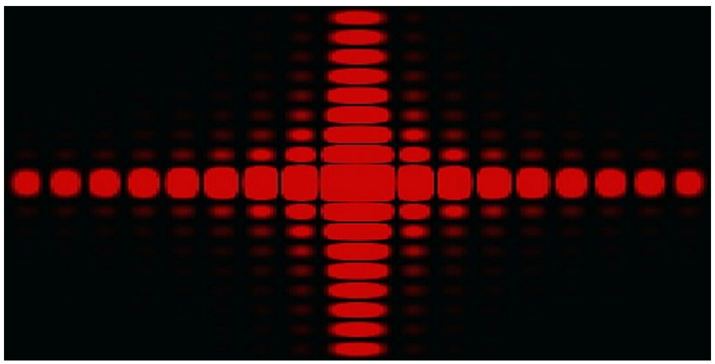
$$I(\theta) = I_0 \left(\frac{2J_1(ka\sin\theta)}{ka\sin\theta} \right)^2$$

k is wavenumber, J_1 is Bessel function

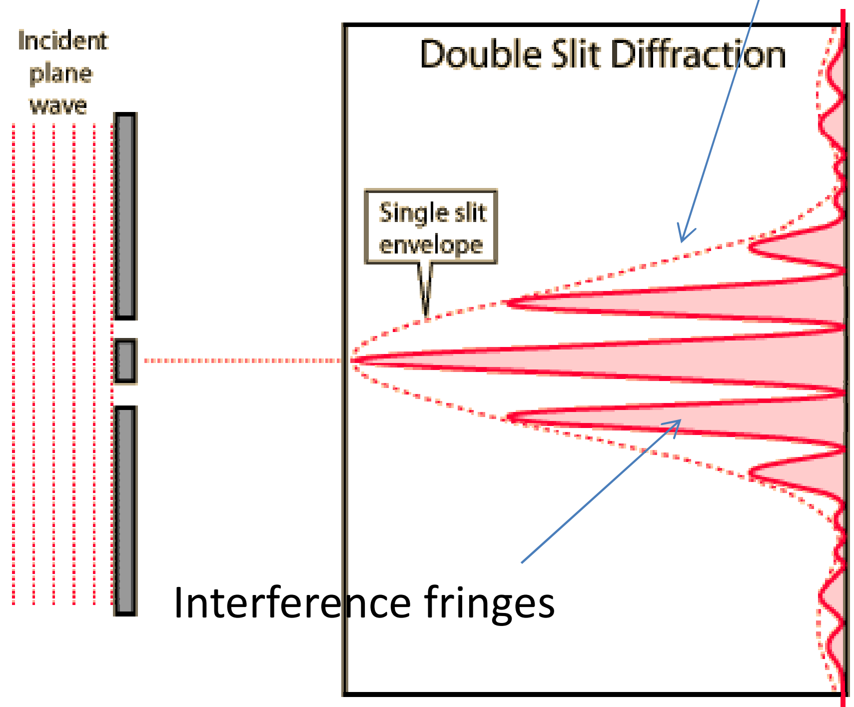
Other Types of Patterns

Airy function

- Rectangular aperture
- Young's Double slit



Spreading is inversely proportional to the size of the slit relative to the wavelength of the light. It is also perpendicularly directed.



Many Slit Diffraction

- As you add more slits, you need to account for extra interference terms
- The single slit diffraction envelope remains the same

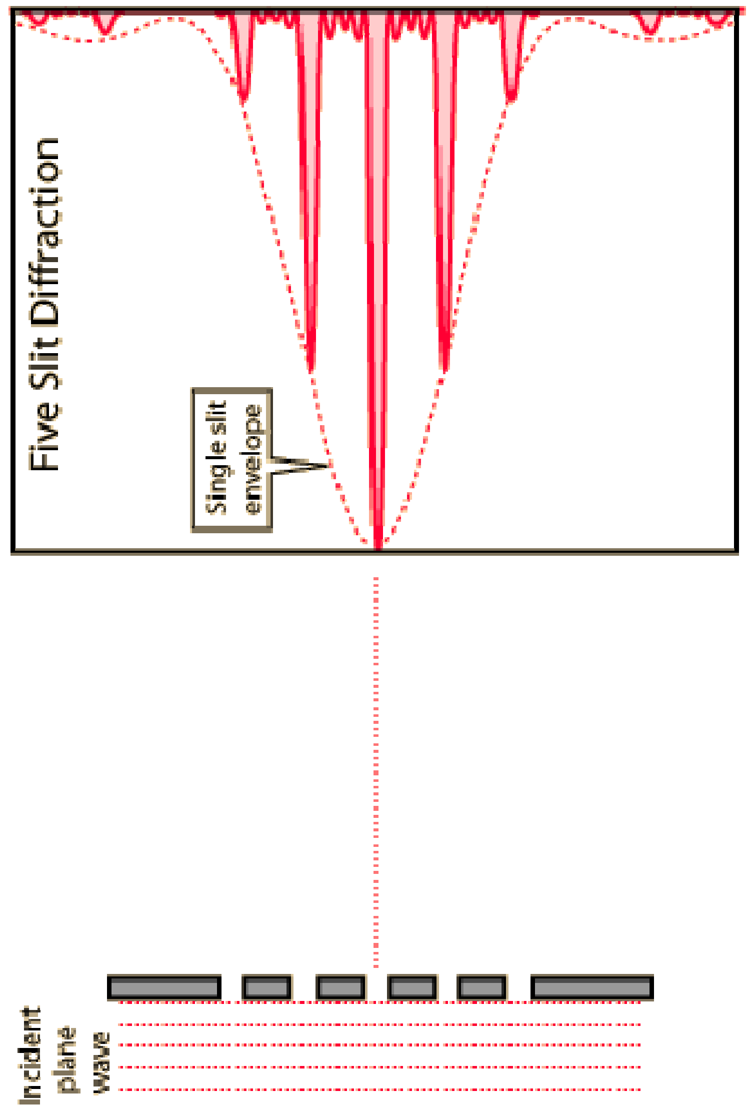
diffraction — interference

$$I = I_o\left(\frac{\sin\beta}{\beta}\right)^2\left(\frac{\sin N\alpha}{\sin\alpha}\right)^2$$

$$\beta = \frac{1}{2}kb\sin\theta$$

$$\alpha = \frac{1}{2}ka\sin\theta$$

b is slit width, a is slit separation, k is the wavenumber



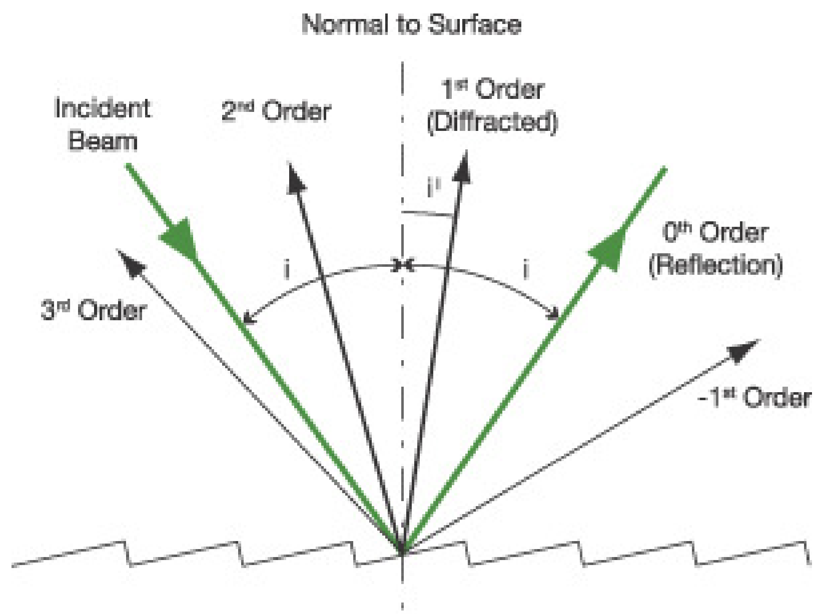
Notes about Many Slit Diffraction

- If a "zero" from the diffraction envelope lands on top of a "max" from the interference, you may appear to have missing orders in your intensity plot
  - See discussion pages 286-288 in your text
- The many-slit diffraction is the basis for an often used device for resolving spectral lines called a diffraction grating

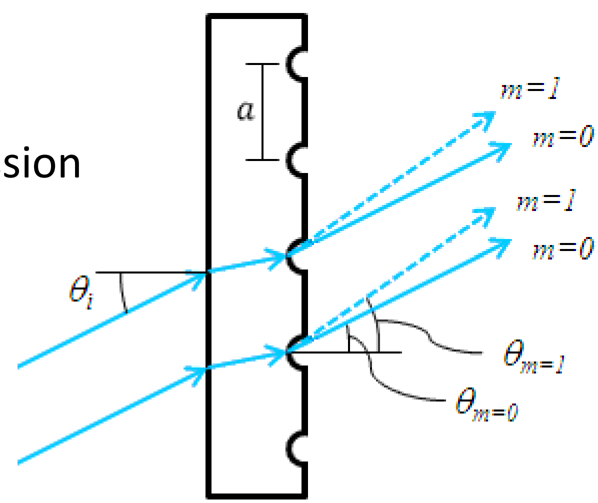
Diffraction Grating

- Gratings use either reflection or etching in a substrate to produce “multiple slits”
- Dispersion and spectral resolution use the same defn. as with prisms – table 12-1
- Orders, m, go to different locations on your screen

Reflection grating



Transmission grating



Grating Equation and Blaze

- For normal incidence the equation is simple
- We need an equation for arbitrary incidence angle, Θ_i, on the grating

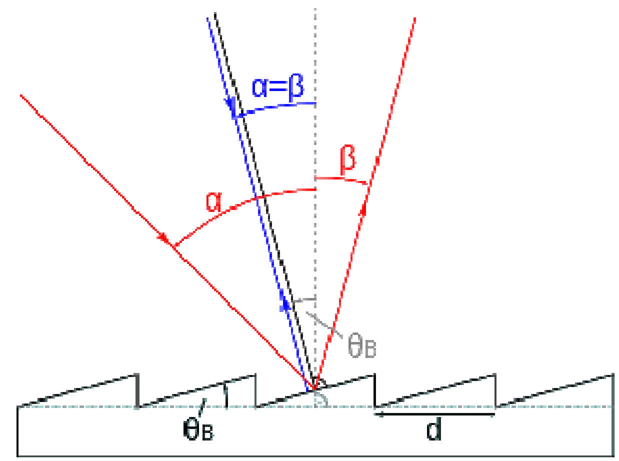
$$d\,(\sin\theta_i + \sin\theta_m) = m\lambda$$

Θ_m is order maximum angle, m is order and d is separation between grooves

- Grating blaze is an angle you add to the face of the grating to improve its efficiency
- A blazed grating works best for a particular waveband and order, m
- Littrow is a term used when the incident light is brought in along the groove face normal

Blazed Grating con't

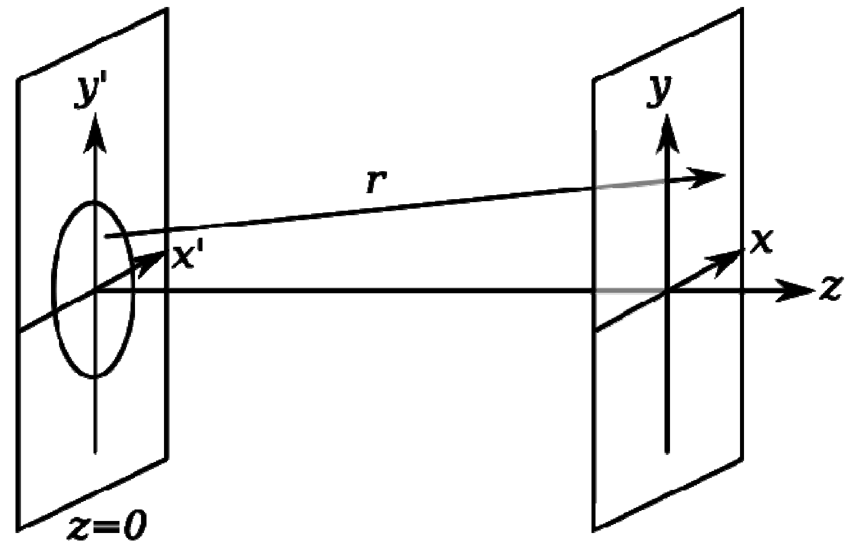
- Grating equation for Littrow configuration is a little simpler
  - d is line spacing
  - α is incidence angle
  - β is diffraction angle
  - m diff. order
- Can have a blaze angle on a reflection or transmission grating
- Often referred to as echelle gratings if the blaze angle is > 45 deg.



$$d\,(\sin\alpha + \sin\beta) = m\lambda$$

Fresnel Diffraction

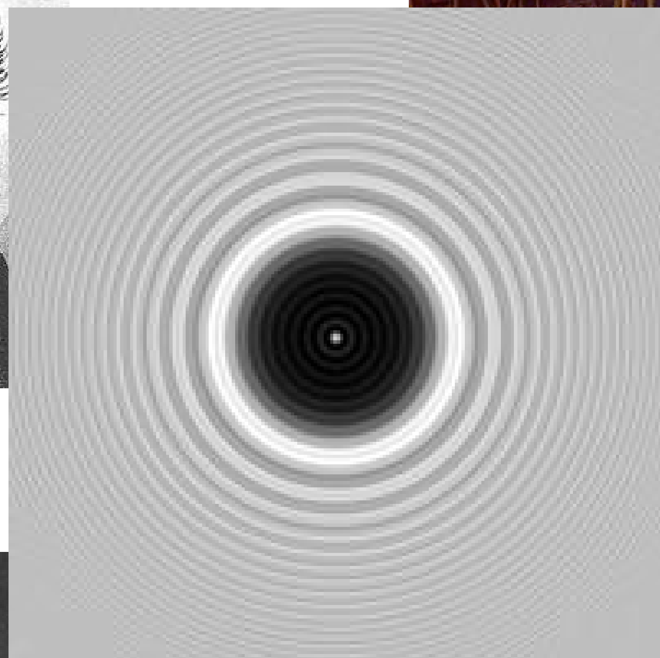
- Near-field diffraction
- Dealt with by calculating the detailed E field using a Fresnel-Kirchoff integral and usually not with "nice" geometry
- Can be calculated with convolution and a FT
- Can be approximated by considering phase zones

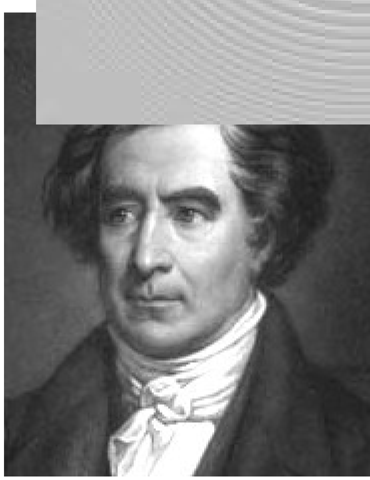


$$E(x,y,z) = \frac{z}{i\lambda} \iint_{-\infty}^{+\infty} E(x',y',0)\frac{e^{ikr}}{r^2}dx'dy'$$

aperture

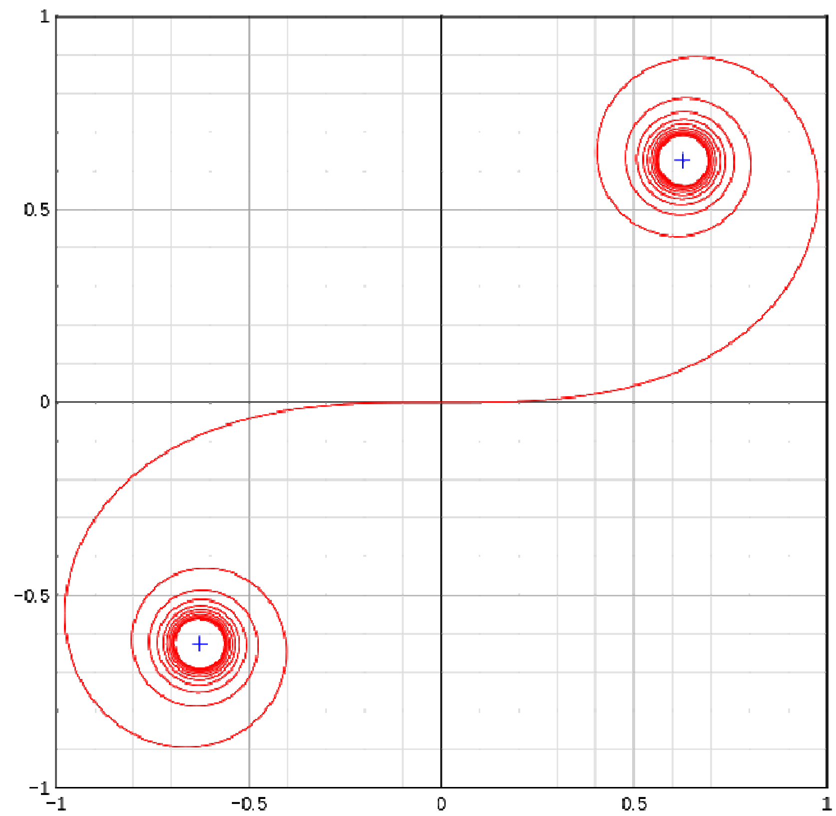
Story of Fresnel, Arago and Poisson





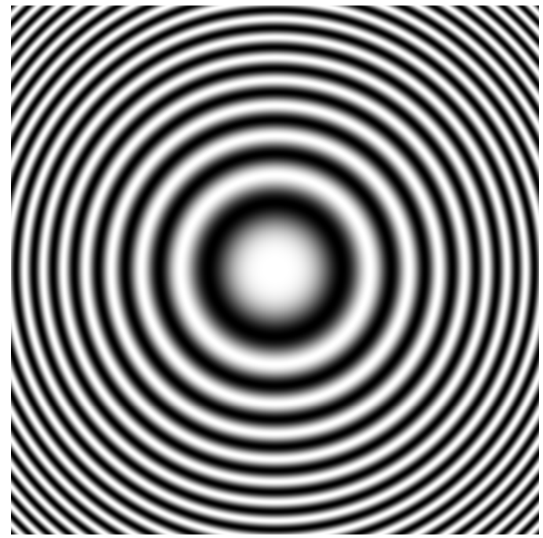
One Way to Do This

- Using an Euler or Cornu Spiral
- There are examples shown for you in sections 13.7 to the end of that chapter in your text
- If you ever find yourself in need of doing this, consult an expert as the math is not for the faint of heart



Fresnel Zone Plate

- Device manipulating phase in an aperture to get it to perform like a lens or other optic
- It is in some sense the FT complement of the Poisson spot
- Huge advantage of being compact and light weight
- Used in situations where "glasses" are hard to manufacture



Applications

- Non-destructive testing of optics or edges for smoothness on wavelength scale
- Fresnel optics
- Use in precision controlled systems and for microscopic cutting
- New types of diffraction gratings
- Changes in seismic structure of ground at radar wavelengths
- Laser speckle applications – beam shaping
- Scattering experiments