

Dynamo and Hydromagnetic Liquid Metal Experiments

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Abstract. Magnetic dynamo experiments are being performed in many laboratories throughout the world. The most successful have been at Karlsruhe and Riga where liquid sodium has been forced in constrained flows and produced predicted positive dynamo gain. The unconstrained flows are being investigated at Wisconsin, Cadarache, and Maryland. These experiments have not reached the critical values of the magnetic Reynolds number, R_m , necessary for dynamo gain, despite the positive predictions of laminar flow theory. Experiments in understanding the relationship of turbulence to MHD are being performed in addition to these at Perm, Princeton, and Swarthmore. A naturally constrained dynamo, using angular momentum gradient, Couette flow, to suppress turbulence is being attempted at New Mexico Tech. Flow constrained by ridged walls is likely to have less turbulence than unconstrained flow where the criterion of constraint is the degree of suppression of the Helmholtz shear flow instability. The study of MHD in conjunction with fluid turbulence has become of major importance in dynamo experiments as well as to future dynamo theory.

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1. Introduction

The need for a magnetic dynamo to produce and amplify the immense magnetic fields observed external to galaxies and in clusters of galaxies has long been recognized. Similarly the magnetic fields in stars leading to the ubiquitous "activity", x-ray and visible flares, as well as planetary fields, and our own earth's magnetic field all require the conversion of free energy, near universally mechanical energy, into magnetic energy. (By way of comparison it is amusing to note that the power required to sustain the earth's field, $\sim 10^5$ watts, is no more than that of a large jet engine yet the dynamo in the massive black hole accretion disks must be larger by a factor of $\times 10^{34}$.) In this context that the dynamo converts mechanical energy into magnetic energy, we specifically exclude other interpretations of a dynamo such as the reverse field plasma pinch, spheromac, and radio lobe formation from being described as a dynamo. These phenomena indeed convert toroidal to poloidal flux, "flux-conversion" (Sovenec CR, Finn JM, and del Castillo-Negrete D 2001), while dissipating a fraction of the magnetic energy, but this dissipation of magnetic energy is the opposite of the production of magnetic energy from a source of mechanical energy. Indeed the reverse field plasma pinch, spheromac, and radio lobe all require a dynamo first (or a chemical battery) in order to create

their magnetic fields in the first place. Furthermore a dynamo is presumed to produce, (either replace or increase) the total magnetic flux, neither of which occurs in the still poorly understood phenomena of flux conversion.

With this definition of dynamo, the theory of kinematic magnetic dynamos has had a long history and is a well developed subject by now. There are numerous monographs and review articles devoted to the magnetic dynamos in astrophysics, some of which are: (Parker 1955, 1979; Moffatt 1978; Stix 1975; Cowling 1981; Roberts 1992; Childress 1990; Zeldovich 1983; Priest 1982; Busse 1991; Krause 1980; Biskamp 1993; Mestel 1999; Gailitis et al. 2000.) Hundreds of papers on magnetic dynamos are published each year. Three main astrophysical areas, in which dynamos are involved, are the generation of magnetic fields in the convective zones of planets and stars, in differentially rotating spiral galaxies, and in the accretion disks around compact objects. The possibility of production of magnetic fields in the central parts of the black hole accretion disks in AGN has been pointed out by Chakrabarti 1994 and the need and possibility for a robust dynamo by Colgate 1997. In the last few years positive gain dynamos have been created in the laboratory in the Karlsruhe experiment (Stieglitz 2001) and in the Riga experiment (Gailitis 2000, Gailitis 2001). (Here we use *gain* to mean the positive exponentiation of magnetic flux by a feedback mechanism from poloidal to toroidal and back again,

as opposed to the *amplification* of a given, bias flux when it is converted into a larger orthogonal one with no increase of the original bias flux.) These experiments prove beyond reasonable doubt that the dynamo equations of magnetohydrodynamics are an accurate description of Nature. Why then do we still need to understand and experimentally prove a dynamo in the laboratory? Three large experimental efforts to produce dynamo gain in the laboratory using less constrained flows, the von Karman or Dudley James flow, (Dudley, M. L., & James, R. W. 1989) although initially predicted by numerical simulation to produce dynamo gain, have in fact failed to do so. There is emerging concern that turbulence, not included in the initial laminar analysis, may be interfering with the predicted dynamo gain. As Fausto Catanio (Univ of Chicago) so eloquently put it. "Can a dynamo exist despite turbulence not because of it?"

The plan of this paper is to enumerate the principle liquid metal experiments of the world and to briefly discuss their salient features in section II. The difference between constrained and unconstrained flows will be described in section III along with a brief heuristic interpretation of the large difference in the magnitude of turbulence, a factor of ~ 100 , expected between constrained and unconstrained flows. This large difference will be interpreted as the explanation for the failure of dynamo excitation in the experiments using unconstrained flows. In Section IV the natural constraints to turbulence of several naturally occurring astrophysical flows, such as angular momentum in accretion disks and entropy gradients in stellar atmospheres (the region of the radiative zero solution) will be used to justify (constrained) astrophysical dynamos. The attempt at the simulation in the laboratory of one such flow will then be discussed.

2. Liquid Metal Experiments

Some of the liquid metal MHD experiments of the world (some, only because there must be others of which I am unaware), are listed below: ([AS] = axi symmetric hydrodynamic flow)

1. ICM Perm, Russia (MHD, Pomeranchenko, high rotation constrained shear sodium dynamo, [AS])
2. Riga, Latvia (Driven, constrained counter current single helical cylindrical sodium dynamo flow, 1m, [AS], works)
3. Karlsruhe, Germany, (Driven helical counter current flow dynamo in multiple tubes, 1m, [non-AS], constrained, sodium, works.)
4. Princeton Plasma Lab, N.J., USA (0.3m gallium experiment to demonstrate MRI, minimizing end-wall, Ekman layer driven turbulence and stable Couette flow.)
5. Grenoble, Lyon, Caderache France (10, 30, 60 cm dynamos, Von Karman, [Dudley-James], sodium and gallium unconstrained flows [AS])
6. U of Maryland, (Dudley-James dynamo flow in 1m sphere, Thermal convection driven turbulence, MRI experiments, unconstrained sodium flow [AS].)
7. U of Wisconsin, Madison. (Dynamo, Dudley-James, MRI, 1m, unconstrained, sodium flow [AS].)
8. Swarthmore, Penn. (0.15 m Couette flow experiment in sodium to demonstrate enhanced resistivity by turbulence [AS])
9. NMTech, LANL, (0.6m $\alpha\omega$ Dynamo, [non-AS] liquid sodium flow constrained by stable Couette flow and helicity derived from driven plumes, also MRI experiments.

2.1. Perm

The Perm experiment (Noskov V. et. al. 2004) uses a rapidly rotating torus filled with liquid sodium. The sudden breaking of the rotation of the toroidal boundary leads to a high velocity shear within the fluid by fluid friction with the walls. This experiment will explore the ω phase of dynamo amplification, namely the amplification of the flux orthogonal to the applied flux and parallel to the shear.

2.2. Riga and Karlsruhe, the Successful Dynamos

The Riga and Karlsruhe experiments both use counter current liquid sodium helical flows that are separated by ridged pipe wall(s). In the Riga experiment (Gailitis, A., Lielausis, O., Dement'ev, S., et al. 2000 & Gailitis, A., Lielausis, O., Platcis, E., et al. 2001) the helical flow is driven by one impeller and the rotation lasts for the injected as well as return flow. In the Karlsruhe experiment (Stieglitz, R., & Müller, U. 2001) the helical flow is maintained within many relatively long pipes with return flow. There are numerous, ~ 30 , nested pipes within which a helical vane maintains the helical flow. The pumping of the flow through the pipes is performed by a pump removed from the dynamo apparatus.

2.3. Princeton

The Princeton Plasma Lab experiment (Goodman, J. & Ji H. 2002) is directed at producing stable cylindrical Couette flow in liquid gallium with the absolute minimum of turbulence for the purpose of observing the growth of the magneto rotational instability, (MRI) in circumstances with the smallest possible shear stress of a turbulent background. Cylindrical Couette flow is ideally stable for $(d\omega/dr) \times (r/\omega) < (-2)$. In a rotating laboratory apparatus end walls drive an Ekman circulation of magnitude $Rm^{1/2}$ which induces turbulence of magnitude $v^2 Rm^{-1/2}$. In order to circumvent this source of turbulence, the end walls are constructed with a velocity or rotation gradient by several graded rotating annuli. In the New Mexico experiment, item #9 this magnitude of turbulence is considered sufficiently (constrained) for dynamo gain at $Rm \simeq 10^7$.

2.4. Cadarache, Maryland, & Wisconsin, the Dudley James -von Karman flows

Three experiments at Cadarache France, von Kraman flow, (M. Bourgoïn, L. Marie, F. Petrelis, et al. 2002), Univ. of Maryland, and Univ. of Wisconsin, Dudley James flow (Lathrup DP, Shew WL, & Sisan DR 2001; Forest, CB et. al.

2002; Nornberg et. al. 2005) are all created within a stationary spherical vessel of roughly a meter in diameter with liquid sodium. The Dudley James or von Karman flow is described as the so-called $t2s2$ flow which in turn is two opposed or face-to-face dipolar flows, a quadrupole poloidal flow with the meridional flow directed inwards towards the common axis and with each flow counter rotating relative to the other. The drive for this flow can be either the axial ejection of fluid towards the opposite poles, driven propellers on axis (the Dudley James flow) or driven by two fluted impellers the von Karman flow, that centrifugally drive a radial and rotating flow at each end of the spherical chamber. This radially forced flow causes the fluid to return to the axis at the midplane or meridional flow in the same fashion as the axially driven Dudley James flow. The flows from each end are counter rotating. It is the shear of this counter rotation across the midplane that gives rise to the production of toroidal field from an initially applied bias or axial field. The ratio of the produced azimuthal or toroidal field to the bias field is called the *amplification* and is significantly smaller in the Wisconsin experimental measurements, 1.1 compared to the predicted value of 1.25 for laminar flow. A larger value of *amplification* is predicted for stable Couette flow, NMTEch, of $\sim Rm/2\pi \simeq 20$.

The salient feature of these unconstrained flow experiments is that they all show large fluctuations of an imposed bias field as if driven by the turbulence inferred from initial velocity measurements. These fluctuating velocities approach up to $1/3$ the mean flow velocity. In Section IV the reduction in Rm due to this implied turbulence is discussed. However, the advantage of the Dudley James - von Karman (DJVK) flow is that it predicts positive dynamo gain at the lowest $Rm \sim 50$, and even less, ~ 43 (Forest, CB et. al. 2002) with an optimized flow in the laminar limit. A further experimental advantage is that it predicts this positive gain inside a stationary vessel using a steady state flow. Currently a calculation of the critical Rm (for positive dynamo gain) using the actual measured flows in water is as high $Rm \simeq 178$ (Nornberg et. al. 2005). So far, probably because of this high critical Rm , none of the three experiments have achieved positive dynamo gain. A larger experiment is proposed to achieve a larger laminar Rm by the Univ. of Maryland, although one worries that the limiting factor may be the growth of turbulence in proportion to the increase in the laminar Rm and hence, to size.

2.5. Swarthmore - Turbulence and Reduced Conductivity

The Swarthmore experiment using liquid sodium in a modest spherical vessel, 0.15 m (Reighard, A.B., & Brown, M.R. 2001) has been built to measure the expected decrease in electrical conductivity expected from turbulent motions in a conducting fluid with $Rm \sim 1$ to 8. The turbulence was driven by thermal convection where, however, the modest velocities of convection of ~ 1 m/s are limited by the small thermal expansion coefficient of sodium. This increase in conductivity, (Krause, F. & Radler, K.-H 1980) is described as:

$$\sigma_{turb} = \sigma_0 / [1 + \mu_0 \beta \sigma_0] \quad (1)$$

The constant β is derived from mean-field electrodynamics assuming isotropic turbulence:

$$\beta \simeq (\tau_{corr}/3) < v_{turb}^2 >, \quad (2)$$

where τ_{corr} is the mean correlation time of a turbulent fluctuation and $< v_{turb}^2 >$ is the mean square fluctuating velocity. Since $\tau_{corr} = L_{corr}/v_{turb}$, then $\mu_0 \beta \sigma_0 = \mu_0 \sigma_0 (L_{corr} v_{turb}/3)$. We then identify $(L_{corr} v_{turb}/3) = D_{turb}$ as a turbulent diffusion coefficient and the turbulent conductivity becomes the original conductivity decreased by the factor $\sigma_{turb} = \sigma_0 / [1 + D_{turb} \sigma_0]$. Since $\sigma_0 = 1/\eta_0$ where η_0 is the original resistivity or equivalently the magnetic diffusivity, we see that the turbulence has added an additional diffusivity to the classical one. In the limit of a large $D_{turb} \gg \eta_0$, then the effective magnetic diffusivity becomes just the turbulent diffusivity, D_{turb} . It is this turbulent diffusivity that ultimately, in this interpretation, dominates the question of dynamo gain in the unconstrained flows.

2.6. New Mexico Tech, a Constrained $\alpha\omega$ Dynamo

At New Mexico Tech a modest endeavor has been underway for a number of years to demonstrate dynamo gain in a configuration, the $\alpha\omega$ dynamo (Parker 1959) that specifically attempts to simulate dynamos in astrophysical circumstances (Colgate, S.A., et. al. 2002). Here low turbulence Couette flow is induced between two co-rotating cylinders, 60 cm and 30 cm diameters at $Rm \sim 120$. The ratio $\omega_1/\omega_2 = 2$ is such as to predict stable Couette flow. However the Ekman flow at the end walls of the cylindrical volume induces a torque (to make up for the flux of angular momentum of the Ekman flow) that is balanced by a weak turbulence in the flow, such that $< v_{turb}^2 > \simeq v_0^2 / Re^{1/2}$ where $Re \simeq 10^7$. The torque and velocity distribution inferred from pressure measurements within the Couette flow, the ω flow, has been measured and agrees with predictions. Therefore the stabilizing influence of the angular momentum gradient of the Couette flow leads to a constrained ω flow. The α or helicity generation depends upon driven plumes that presumably simulate the effects of initial large scale convective elements rising at the base of the convective zone of stars or the result of star disk collisions in the case of the massive black hole accretion disks. The translation, expansion, and rotation of these plumes become a key element in the prediction of dynamo gain (Beckley, HF et. al. 2003). Because of the off-axis, non axis symmetric flow of the plumes, Cowling's anti dynamo theorem therefore allows the possible generation of a unidirectional field or equivalently the generation of large scale, unidirectional magnetic flux. Because of the large rotational shear of the Couette flow, the amplification of an initial poloidal field into a toroidal field should be large, $\sim \times Rm/2\pi \sim 20$, but because the container and hence experimental diagnostics must rotate at a high speed, $\omega_2 \simeq 17.5$ Hz, the experiment has presented unique challenges.

In order to understand these experiments we must discuss the constraints and symmetries of the fields and flows. Presumably the over-arching question of dynamo research is finding an experimentally provable explanation for the origin

of the immense magnetic energies and magnetic fluxes of astrophysical phenomena. Although, on the other hand, the endeavor of understanding dynamo theory in the abstract and thus all possible dynamos is an intellectual challenge as deep as any mathematical theorem.

3. Constrained and Unconstrained Dynamo Flows

Constrained and unconstrained flows in dynamo theory can apply to the current or to the fluid flow. Obviously the most constrained configuration for a dynamo is the use of conducting wires with insulation between them so that there is no fluid flow and the topology of the currents can be determined in a pre-arranged fashion, as in an electric motor or generator. Since the combination of conduction and non-conduction is a unique circumstance of human intervention, such constrained currents are unlikely to explain cosmological, stellar, or even planetary magnetic fields. Thus the formation of a dynamo within a conducting fluid by fluid motions alone becomes the challenge of a dynamo. Many laminar fluid flow configurations predict dynamo gain. A gain of greater than unity is therefore the holy grail of success of dynamo experiments, although a demonstration of a reduced decay rate, a decay rate of an applied bias field slower than the stationary fluid case, is indicative of positive dynamo action or proof of principle. Necessarily a positive exponential gain will always lead to a back-reaction, saturated limit, because the exponentiation is only bounded by the back reaction of the generated field upon the flow. Thus in practical terms of an experiment, the dynamic range of exponentiation may be relatively small.

3.1. Fluid and Current Flow Constraints

Constrained and unconstrained dynamo flows instead applies to the fluid motions as opposed to the gradients of conductivity. In order to create a successful dynamo flow, it is very much easier to use ridged boundaries such as pipe walls or vanes to direct the flow into the pattern envisaged for optimum dynamo gain. The source of free energy for the flow can then be well removed from the dynamo experiment itself. Such is the case for the *successful* Karlsruhe experiment where multiple parallel tubes with internal helical vanes and return flow of the sodium simulate the multiple Taylor columns of convection in a rapidly rotating, presumed planetary, flow, i.e., where the rotation period is short compared to the time of convection. (For the earth this ratio is one day/40,000 years. For the sun it becomes 22 days/10 days, and so the Karlsruhe experimental configuration simulates primarily planetary flows, but both experiments are an overarching confirmation of MHD theory.) The *successful* Riga experiment similarly enforces a helically flowing counter current flow. In this case the experiment consists of one large tube with an imposed, driven helical flow at one end and no internal vanes to guide the remainder of the azimuthal flow. The azimuthal flow becomes the surviving fraction of the angular momentum flux imposed at the driven end. In

both flows it is important that the guiding tubes be conducting with small (oxide insulating) surface layers so that the voltage (potential) drop of the large currents flowing across, orthogonal to, the ridged pipe boundaries is small. This represents a significant experimental challenge. In other words if the pipes were made of insulating materials, the dynamo would not work.

3.2. Axial Symmetry

The difference between the two experiments is the strength of the helical drive. In addition the axial symmetry of the single pipe Karlsruhe experiment is more axially symmetric than the multiple nested tubes of the Riga experiment, which give rise to a small, azimuthal, non axial symmetry, whereas the single tube of the Riga experiment is perfectly axi-symmetric. Thus positive dynamo gain in these experiments requires, by Cowlings anti-dynamo theorem, a second source of non axial symmetry other than the flow field itself. This source, as in the Dudley-James, von Karman flows, is the generated field itself, which oscillates in a semi non random and non axi-symmetric fashion. The frequency of the changing fields is determined approximately by the time constant of the magnetic flux diffusion due to the finite resistivity of the conducting fluid itself, i.e., the liquid sodium. This non-symmetric oscillation may be particularly sensitive to turbulence, both for its creation as well as for the consequential enhanced turbulent resistive diffusion.

3.3. Turbulence, the Ridged Wall Constraint and Unconstrained Flow

The common feature of all the predicted positive gain dynamo flow fields is one of shear between two flows and thus within the flow itself. The shear stretches the field and thus does work, $PdV = dx(B^2/8\pi)$. Depending upon the accessible (i.e., the lack of constraints) of fluid instabilities and depending upon the relative times available this same shear does work on the turbulent energy field ρv_{turb}^2 (hereafter ρ , the density, is assumed unity). This turbulent energy field, turbulence, then gives rise to a turbulent diffusion coefficient, $D_{turb} = L_{turb}v_{turb}$ that defeats the magnetic work by allowing the magnetic flux to diffuse relative to the fluid velocity field, i.e., reduces the effective $Rm_{turb} \simeq vL/D_{turb}$ for $D_{turb} \gg \eta_{Na}$. Thus the velocity shear is the source of free energy for creating dynamo gain as well as the source of free energy for the turbulence. Without some constraint on the development of turbulence a positive dynamo gain may become problematic.

The onset of turbulence depends upon the fluid viscosity, ν and fluid velocity, $Re = Lv/\nu$, but the turbulence magnitude, v_{turb}^2 , and diffusion constant, $D_{turb} = L_{turb}v_{turb}$, depend additionally upon the constraints. The difference between D_{turb} in unconstrained and constrained flows may be many orders of magnitude. For example if the gradient of either angular momentum or entropy leads to absolute stabilization of the Kelvin-Helmholtz instability in shear flow the turbulence can be near zero and the Re near infinity as in stars and accretion disks. When walls are present as in

pipe flow the coefficient of friction defines the shear stress at the wall, $\tau_{wall} = C_f v^2$. Since near the wall the turbulent shear stress is independent of the distance from the wall, x , $D_{turb}(dv/dx) \simeq (xv_{turb})(v/x) = \tau_{wall} = C_f v^2$, then $v_{turb} \simeq C_f v$. At high $Re \simeq 10^7$ typical measured values in planar Couette flow are $C_f \simeq 2.5 \times 10^{-3}$ (Physics Handbook). Despite the several approximations the value of the turbulent velocity caused by shear in contact with a wall is so low that $D_{turb} \simeq 2 \times 10^{-3} vL$.

In unconstrained flow on the other hand, as in plane parallel shear between two regions of uniform velocity, analogous to a smoke plume, the mixing half angle determined by the Helmholtz instability is $\sim 1/2\pi$. This angle implies a turbulent diffusion of $D_{turb} \sim (Lv)(1/2\pi)$ or very much larger than in shear flow in contact with a ridged wall. Since the unconstrained diffusion is so much greater than the laminar magnetic diffusion, the value of $Rm \simeq Lv/D_{turb} = 2\pi$. Thus in unconstrained flow the Rm should be small enough such as to inhibit positive dynamo gain. By way of comparison the Ekman layer driven turbulence in the Couette flow experiment leads to a measured value of $C_f \simeq 3 \times 10^{-4}$.

4. Conclusion

The study of MHD in conjunction with fluid turbulence has become of major importance in dynamo experiments as well as to future theory. The current experiments now underway will give deep insight into this new and difficult topic. One notes the importance of these experiments for without them, the importance of flow constraints might have been buried for a future time.

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